-|- Control Systems -|-

9995020392

Subrahmanyam S V
7569653427

① Control System engg. by Nagrath & gopal.

** ② " ——— " - NISE

(IES) ③ Automatic Control System by B.C. Kuo.

(4) Modern Control System - (ogata) (State Space Analysis)

* Transfer fun^ / Block Diag / SFG ① or (2 m)

* Time domain Analysis (Evaluate the performance of System w.r.t. time) ② m.
  - Transient Analysis
  - Steady State Analysis

* Stability
  - Time domain technique → RH④ / RL②
  - Frequency domain technique → B.P③ / NP①

  } ② mark (4)

* Compensators / Controllers.

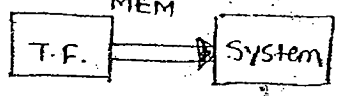
* State space analysis. (2m) (4m)
  (valid for dynamic System, i.e. L/NL/TV/TIV)

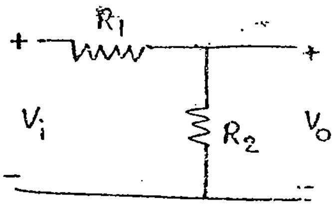
$$T.F. = \frac{1}{S^1+2} = \text{mathematical equivalent model of System}$$

$\boxed{\text{order} = 1}$ ⇓

No. of Storage elements.
No. of time constant

MEM

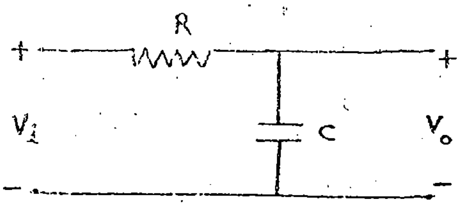




$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1+R_2}$$

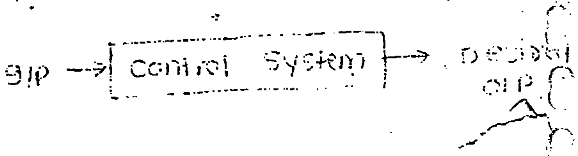
order = 0.

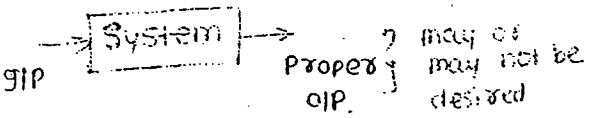
No. of Storage element = 0.



$$\frac{V_o(s)}{V_i(s)} = \frac{1}{SRC+1}$$

- Frequency domain technique give only steady state Analysis not transient Analysis factor (t_r, t_p, t_s etc.)
- Stability is used to find for only CLCS, not for OLCS.





Bulb – Electrical System.
Fan – Electrical System.

A Fan w/o Blade → No Air Flow → Not a System.

A Fan w/o Regulator → System → constant air flow.

A Fan with (Regulator) → Desired o/p → control system.
Controller.

System –

A System is a group of Physical components arranged in a such a way that it gives the proper o/p to given i/p.

* the proper o/p may or may not be desired o/p.

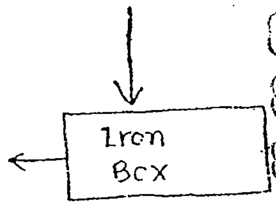
Control System –

A control System is a group of physical component arranged in such a way that it gives the desired o/p by means of control or regulation either direct or indirect method to the given i/p.

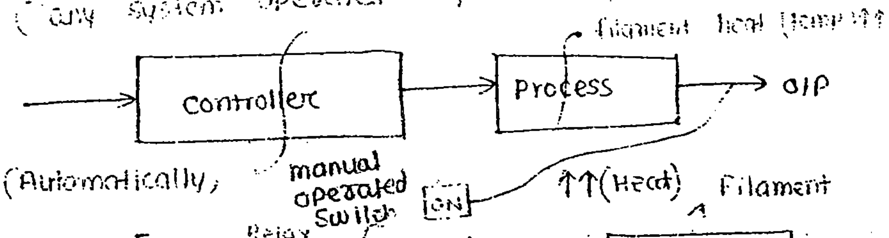
The Control Systems are classified into two ways

Based on controlling system Action.

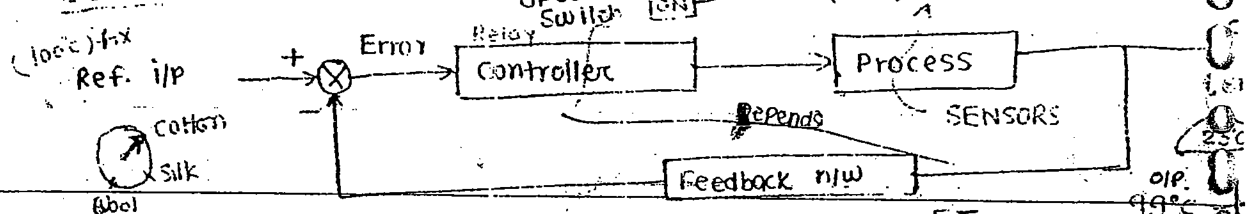
① open Loop Control System.
② closed " " "



O.L.C.S (any system operated by manually)



C.L.C.S (Automatically)



NO traffic

O.L.C.S. (Because of equal duration)

Error (+) = Relay Closed (Normally)

(−) = Open Relay.

E +
+
Error = 0 then Re...

O.L.C.S.

A system in which the controller action independent of the o/p, then it is called O.L.C.S.

Traffic lights, Air coolers, Fan, Tube lights.

any system which have not consist the sensor

C.L.C.S.

A system in which the controller action completely depends on o/p then it is called the CLCS.

For example

any system with Automatic

or

a system which consist the sensor.

Feedback N/W:-

F/b n/w is the property of CLCS. which brings the o/p to i/p and compared with Ref. i/p so that, appropriate control action formed to make the $\boxed{\text{error} = 0}$

i/p = o/p. → System is stable and it gives the desired o/p.

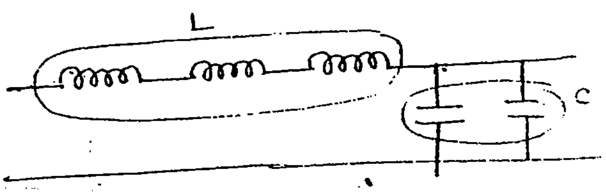
* F/b N/w consist the R, L, C components the max. value of F/b n/w Ratio is 1. the best f/b is unity -ve f/b. The F/b n/w may be a Transducer which converts the Energy from one form to another form

* Transfer Function-

Transfer Function is nothing but mathematical equivalent model of the system.

order of T.F. Represents the no. of storage elements, or no. of time constants.

order of T.F. = 2.



Note - If same kind of elements are connected in series or || then it should be treated as single element.

Linear operator. → α; ↓ Proportional $\int$, diffentiate

T.F. (First Definition)

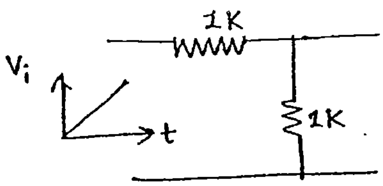
The T.F. of a linear, Time Invarient System is defined as the ratio of Laplace Transform of O/P to L.T. of i/p with all initial cond. are zero. (to maintain Linearity)

$$T/F. = \left.\frac{L[O/P]}{L[I/P]}\right|_{Ii=0}$$

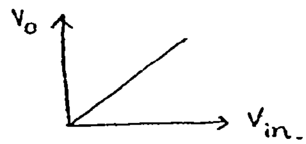
Linear Time Invarient — (R, L, C ckt.)

↓

T.F. char.



$V_o = V_i \times 0.5$

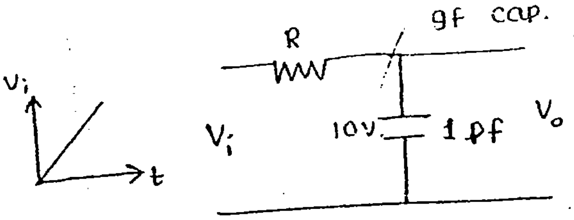


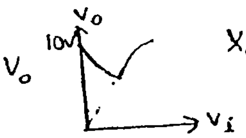
the LTI System is nothing but R, L, C ckt. because R, L, C components gives the Linear Transfer characteristics and R, L, C component values not changes w.r.t. time.
in the T.F. Analysis, the initial condition must be zero to get the Linear transfer characteristics.

T.F. (Second definition)

The T.F. of a linear time invarient system is defined as L.T. Impulse Response with all initial Conditions are zero.

If cap. has initally 10v.





$$X_c = \frac{1}{2\pi f(10^{-12})} = \frac{10^{12}}{2\pi f}$$

= large value

= treat as o.

Impulse → System. → Impulse Response

$$\frac{C(s)}{R(s)} = \frac{1}{s+2} \qquad c(s) = \frac{1}{s+2}$$

$R(s)$ = 1

The Insignificant Pole Should be Converted Into T.C. form and then neglected.

Find the equivalent T.F. to the Following,

$$\frac{C(S)}{R(S)} = \frac{1}{(S+1)\,(S+10)\,(S+100)}$$

$\tau = 1 \quad \tau = 0.1 \quad \tau = 0.01$ Sec.

$$= \frac{1}{(S+1)\;10\,(1+0.1S)\;100\,(1+0.01S)}$$

$$= \frac{0.001}{S+1}$$

$\left.\begin{array}{l}\text{System Stability} \\ \text{System Response} \\ \text{system T.C.}\end{array}\right\}$ Poles are Considerd.

System T.C. — "to draw the Response"

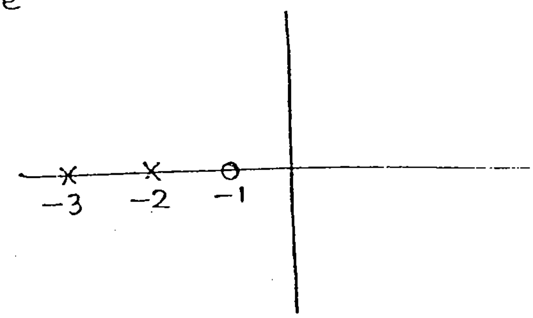
$$\frac{C(S)}{R(S)} = \frac{(S+1)}{(S+2)\,(S+3)}$$

$r(t) = \delta(t) \qquad R(S) = 1$

$$= \frac{-1}{S+2} + \frac{2}{S+3}$$

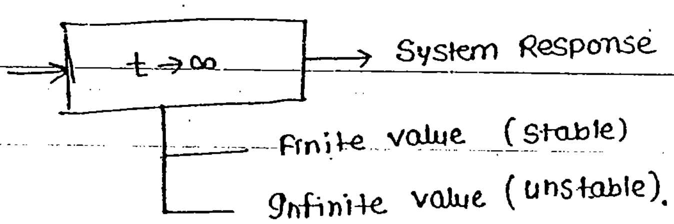
$\downarrow$ I.L.T.

$$c(t) = (-1)\,e^{-2t} + 2\,e^{-3t}$$



* System Response consist only Poles term., No zeroes Response terms are exist So System Stability depend on Pole.

While Finding System Response, System Stability, System T.C. Consider only poles but not zero's because the System Response consist only Poles Response term no more zeros Response terms are Present in the Response.



System Response ⟹ Impulse Response

$r(t) = \delta(t) \qquad R(s) = 1$

$$C(S) = \frac{1}{(S+1)\,(S+10)}$$

$$= \frac{1/9}{S+1} - \frac{1/9}{S+10}$$

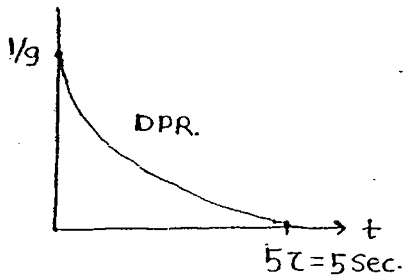
System Res. → $c(t) = \underbrace{\tfrac{1}{9}\,e^{-t}}_{\text{dominant Pole Response}} - \underbrace{\tfrac{1}{9}\,e^{-10t}}_{\text{Insignificant Pole Response}} \approx \frac{1}{9}\,e^{-1.t}$

Note → exponential powers are nothing but Real part of pole locations.

the standard form of first order system.

$$= \frac{K}{S\tau + 1}$$

its System Response $= K\,e^{-t/\tau}$



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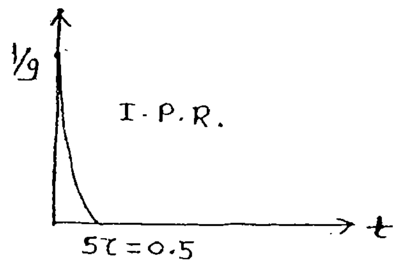

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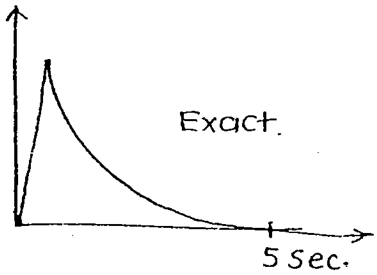

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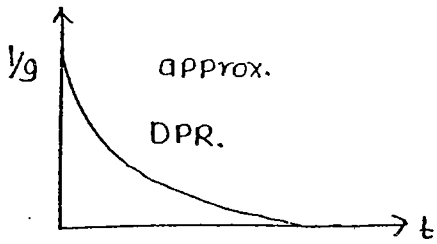


$$\frac{C(S)}{R(S)} = \frac{1}{(S+1)\cdot(S+10)}$$

$$= \frac{1}{10\,(S+1)\,(1+\cancel{0.1S})} \quad \text{Neglect}$$

$$\approx \frac{0.1}{S+1}$$

$c(t) = 0.1\,e^{-1.t} \approx \frac{1}{10}\,e^{-1.t}$

3. The Impulse Response is also called System Response, Natural Resp... Free forced Response because Impulse Response consist only System parameter. (K and τ). { No. i/p term present in the Response). If the i/p Signals are

Step
Ramp
Parabolic

then System Response called Forced Response.

Basics –

The Standard form of System is represented in the form of OLTF i.e.

Representation called Time Const. Form

$$\frac{C(s)}{R(s)} = G(s) = \frac{K(1+s\tau_1)(1+s\tau_2)\ldots\ldots}{s^{n}(1+s\tau_a)(1+s\tau_b)\ldots\ldots}$$

$s^n \rightarrow$ Type - n System.

Type $\rightarrow$ no. of Poles at origin

order $\rightarrow$ no. of poles in the S-Plane or T.F.

$K \longrightarrow$ System gain
$\tau \longrightarrow$ Time Constant.
} System parameter.

Find the System gain and type and order. So the given system

$$\frac{C(s)}{R(s)} = \frac{10(s+5)^2}{s^3(s+2)^3(s+10)} \quad \text{— (Pole zero Form)}$$

$$= \frac{\cancel{10} \times 5^2 (1+0.2s)^2}{2^3 \times \cancel{10}\, s^3 (1+0.5s)^3 (0.1s+1)}$$

$$= \frac{(25/8)(1+0.2s)^2}{s^3(1+0.5s)^3(1+0.1s)}$$

$$K = \frac{\text{Numerator Const.}}{\text{deno. Constant}} = \frac{10 \times 5^2}{2^3 \times 10} = 25/8$$

Type - 3 order = 7

Characteristic Eq. $\Rightarrow$

If deno. of T.F. make =0. then It gives the char. equation.
the char. eq. gives the System Behaviour.
The Roots of char. eq. are nothing but poles.

Pole – Pole is nothing but -ve of inverse of System time Constant at which magnitude of T.F. is ∞.

$$S_p = -1/T_a, \; -1/T_b \qquad |T.F.| = \infty$$

zero – zero is nothing but negative of inverse of System T.C. at which magnitude of T.F. is zero.

$$S_z = -1/\tau_1, \; -1/\tau_2 \qquad |T.F.| = 0.$$

The Pole affect the System stability and System Response but not the zeroes.

Time Constant –

Time const. gives the system behaviour If the T.C. is very² Large then System is called Slow Response System, because it takes Large time to reach the steady state.

Practically, any System takes 5 time Const. to reach S.S.

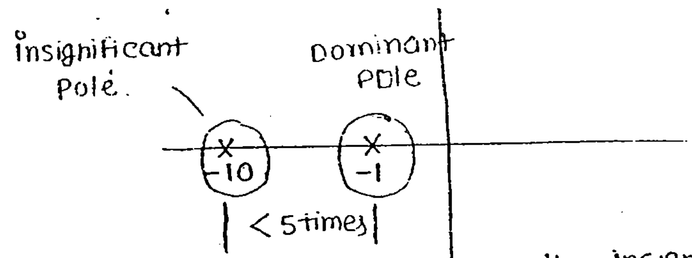
$$\tau = \frac{-1}{\text{Real part of dominant pole}}$$

$\tau \uparrow\uparrow$ – Slow Res – Large time

* Find equivalent First order System to given System.

$$\frac{C(S)}{R(S)} = \frac{1}{(S+1)\,(S+10)}$$

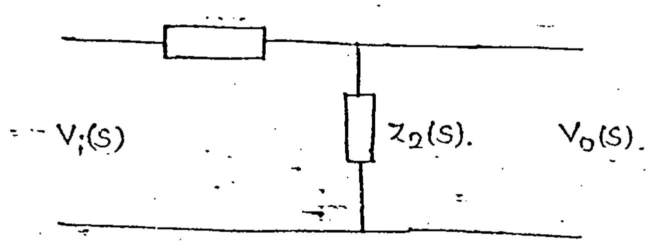
Find the System time Constant through the above System.



Insignificant pole →

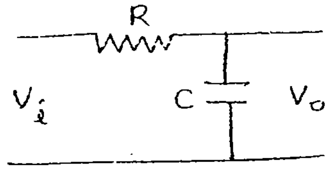
The pole which lies in the left most side. the insignificant Pole Should have Smallest Time Constant., whereas the dominant Pole have largest T.C. the Insignificant pole T.C. must be less than 5 times than dominant pole T.C.

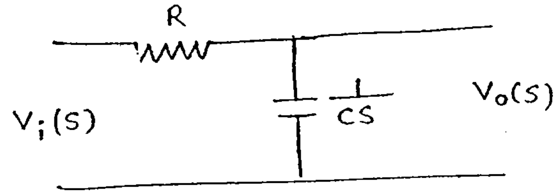
The Insignificant poles are neglected because even If The insignificant Pole neglected there is no much change in System Response.



$$\frac{V_o(S)}{V_i(S)} = \frac{\text{Impedence across O/P}}{\text{Impedence - total}} = \frac{Z_2(S)}{Z_1(S)+Z_2(S)}$$

Find the T.F. to the given electrical, locate the poles in the S-plane and find the System Response





$$\frac{V_o(S)}{V_i(S)} = \frac{1/CS}{R+1/CS}$$

$$= \frac{1}{\cancel{CS}} \times \frac{\cancel{CS}}{SCR+1}$$

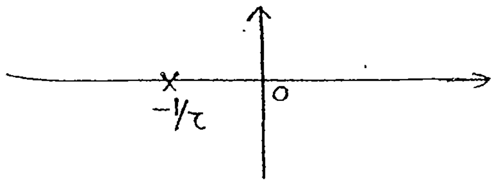
$$= \frac{1}{SCR+1}$$

$$= \frac{1}{S\tau+1}$$

$V_i(t) = \delta(t)$

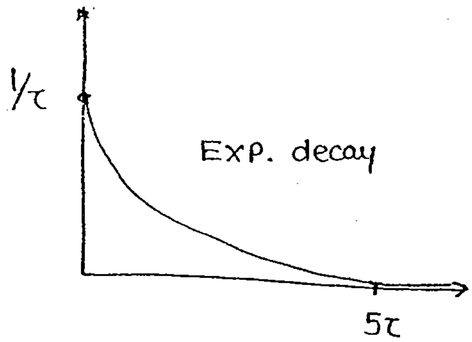
$V_i(S) = 1$

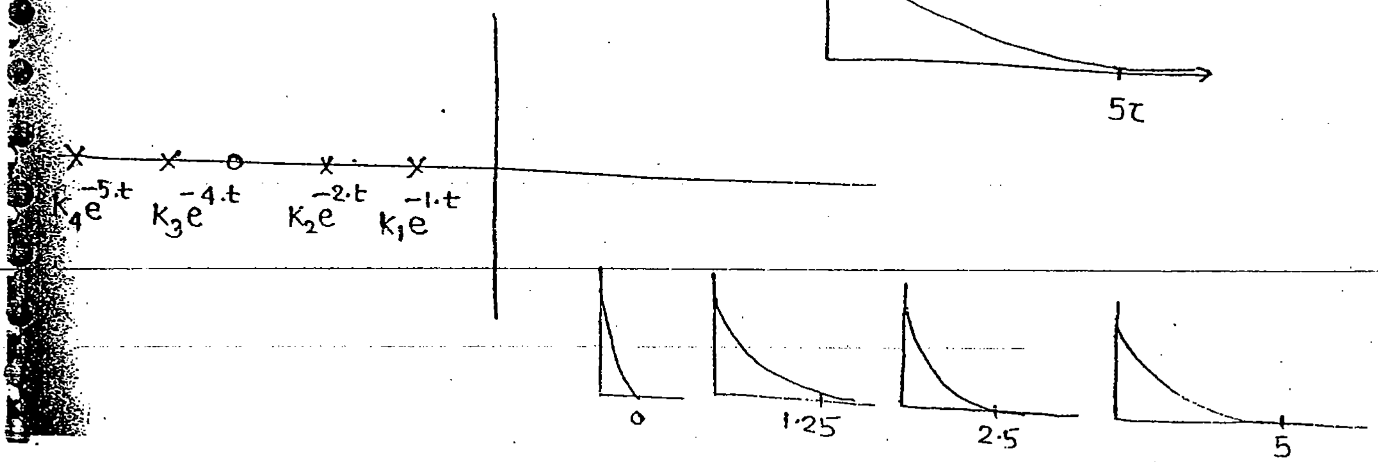
$$V_o(S) = \frac{1}{\tau\,(S+1/\tau)}$$



* *
$$\boxed{V_o(t) = \frac{1}{\tau}\,e^{-t/\tau}}$$

* System is Stable





Note – whenever many ...
Real axis at different locⁿ then the System Response is exponential decay, irrespective of Position of zeroes and system is stable

Addition of Pole -
By Selecting proper value of R, L, C component connected in either Forward Path (Series) or Feedback Path (Parallel)

Movement of Pole → Changing the component Value of R, L, C

If Pole moves towards the origin then we have to check them R and C value

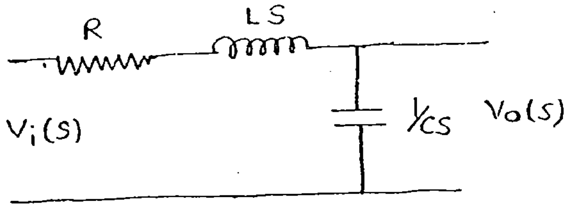
Addition of Poles or zero's
It is nothing but connecting a R, L, C n/w to the system either in series or parallel.
Connecting a R, L, C circuit to the system in series is nothing but connecting in Forward Path.

Absolutly Stable System -
The System is stable for all the values of system parameter like K from 0 to ∞.

Conditional Stable System -
The System is stable for certain Range of System parameters.

Q. Find the T.F. to the given R, L, C Ckt. locate the poles in the S-plane by considering $R = 0\Omega$ $L = 1H$, $C = 1F$.

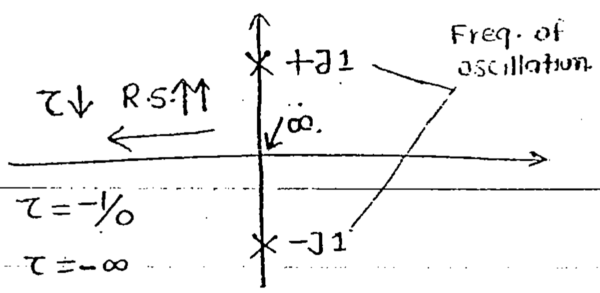
$$\frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R + SL + 1/cs}$$



$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 LC + SCR + 1}$$

$$= \frac{1}{s^2 + 1}$$

$$\boxed{V_o(t) = \sin t}$$



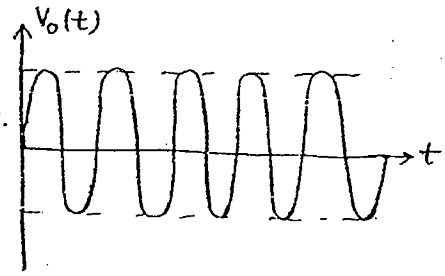
Non-repeated pole on jω axis, system always marginal stable

* in a complex conjugate pole, the Real part of the pole locⁿ always give the system T.C. and Imaginary part of pole locⁿ gives the Frequency of oscillations.

System Response $V_i(s) = 1$.

$V_o(t) = \sin t$ (const. amp. and freq. of oscillation)

* Whenever the poles on Imaginary axis which are non-repeated then system Response is Constant amplitude and Freq. of oscillation which are called undamped oscillation, or Natural freq. of oscillations and system becomes the marginal stable.



Undamped

* Repeat the above problem by considering $R = 1\Omega$ $L = 1H$, $C = 1F$.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + 1}$$

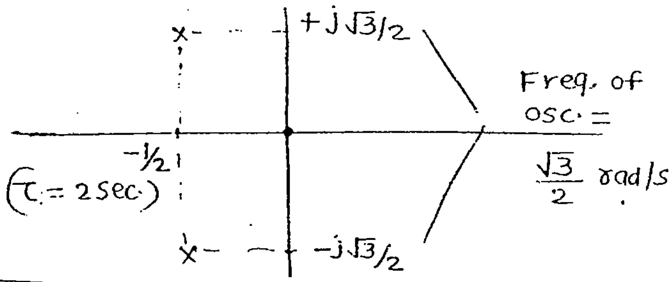
$\omega_n = 1$

$2\zeta\omega_n = 1$

$\boxed{\zeta = 0.5}$

$s^2 + s + 1 = 0$

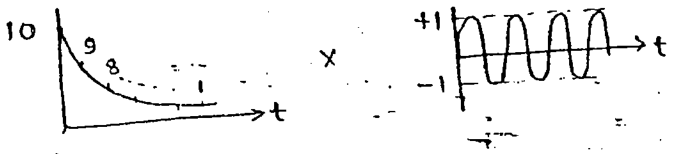
$$s = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}j}{2}$$

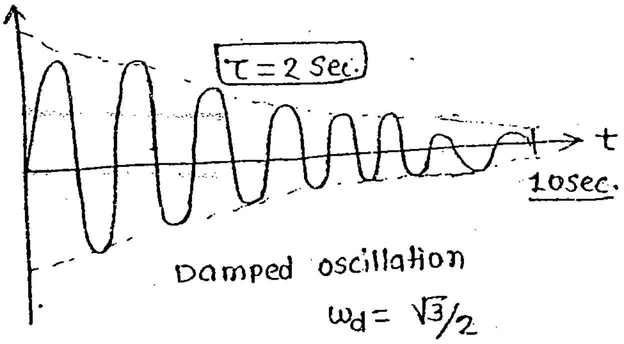


$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(\underbrace{s+\tfrac{1}{2}}_{a} - \underbrace{j\tfrac{\sqrt{3}}{2}}_{b}\right)\left(\underbrace{s+\tfrac{1}{2}}_{a} + \underbrace{j\tfrac{\sqrt{3}}{2}}_{b}\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{2/\sqrt{3} \cdot \left(\sqrt{3}/2\right)}{\left(s+\tfrac{1}{2}\right)^2 + \left(\sqrt{3}/2\right)^2}$$

$$V_o(t) = 2/\sqrt{3}\ \underbrace{e^{-(1/2)t}}_{\text{exp. decay}}\ \underbrace{\sin\left(\sqrt{3}/2\right)t}_{\text{f.o.o.}}$$



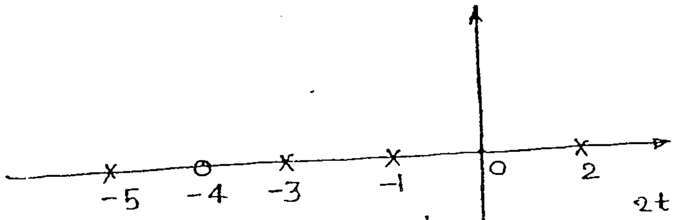


Damped oscillation

$\omega_d = \sqrt{3}/2$

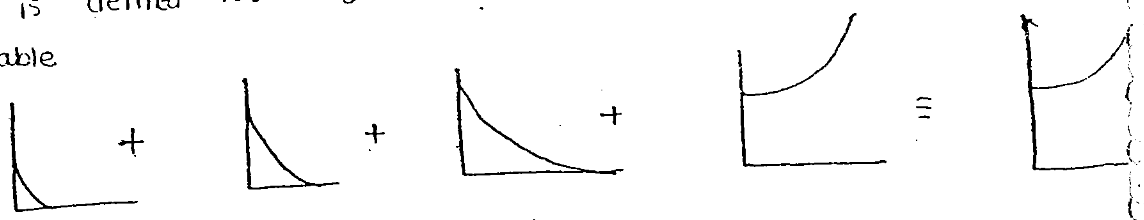
* whenever the poles are complex conjugate in the left half S-plane then system Response is exponential decay frequency of oscillation which are called damped oscillation.
* Any system which produced the damped oscillation called the <u>underdamped system.</u> in this case system is stable.

* Find the time constant and the system Response to the given poles locations in the S-plane

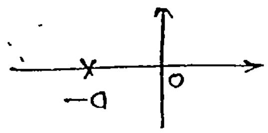


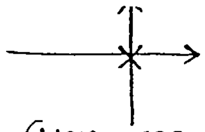
$K_3 e^{-5t}$ $K_2 e^{-3t}$ $K_1 e^{-t}$ $K_4 e^{2t}$

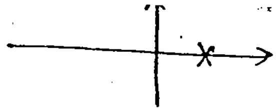
* T.C. is defined for only stable system, the above system is unstable



* whenever one or more poles lies in the Right Half of S-plane remain many poles and zero lies in Left Half S-plane then system Response is exp. Rise to ∞. and system becomes unstable





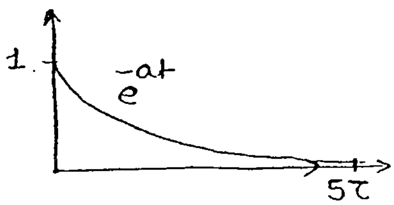


(Non-repeated pole on jw axis System is marginal Stable)

① T.F.
② System Response
③ draw the system Response
④ Stability

ⓐ $c(s) = \frac{1}{s+a}$.

$c(t) = e^{-at}$.

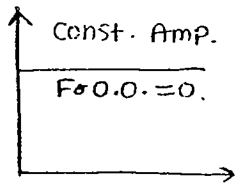


(Stable)

System Response Follow the i/p then system

ⓑ $c(s) = \frac{1}{s}$

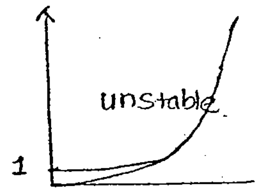
$c(t) = u(t)$



(marginal Stable)

ⓒ $c(s) = \frac{}{s}$.

$c(t) = e^{at}$

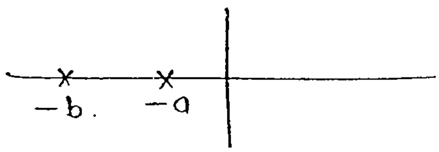


(System Respons moves away from i/p)

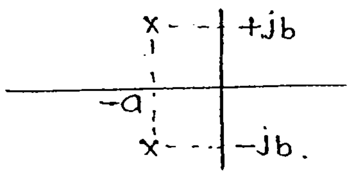
$c(s) = \frac{1}{(s+a)(s+b)}$

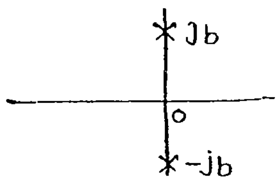
$c(s) = \frac{1}{s^2}$

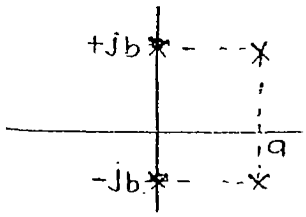
$c(s) =$



Complex conjugate pole







$c(s) = \frac{1}{(s+a+jb)(s+a-jb)}$

$= \frac{1}{b} \frac{b}{(s+a)^2 + b^2}$

$c(t) = \frac{1}{b} e^{-at} \sin bt$

$c(s) = \frac{1}{(s+jb)(s-jb)}$

$= \frac{1}{s^2+b^2}$

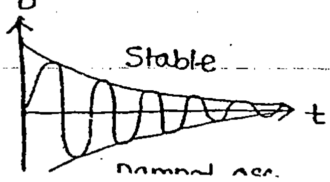
$c(t) = \frac{1}{b} \sin bt$

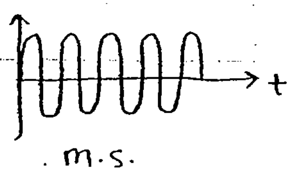
$c(s) = \frac{1}{(s-a+jb)(s-a-jb)}$

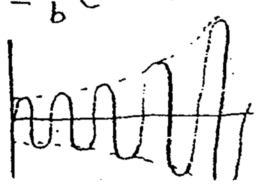
$= \frac{1}{(s-a)^2 + b^2}$

$c(t) = \frac{1}{b} e^{at} \sin bt$

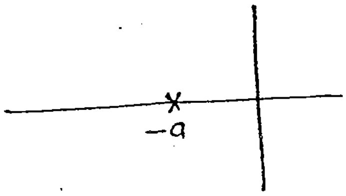
Fold with Respect to 0.







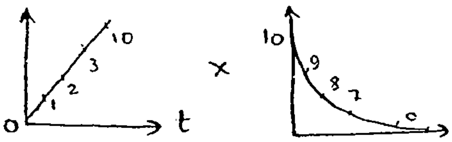
$c(s) = \frac{1}{(s+a)^2}$



$c(t) = t\,e^{-at}$

$c(t) = \frac{t^2}{2}\,e^{-at}$ $\quad \frac{1}{(s+a)^3}$

No change in exp. term.



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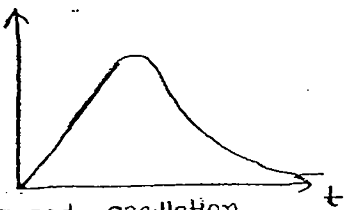
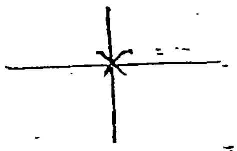


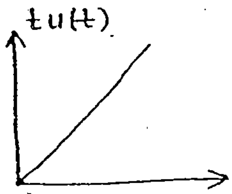
* one damped oscillation

larger value of t → (exponential term dominated)

lower value of t

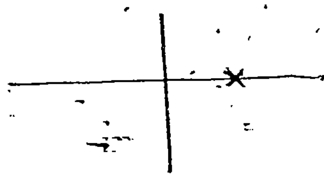


$c(t) = t\,u(t)$.

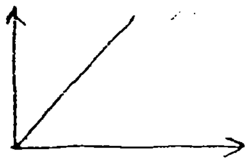


unstable

$(s-a)^2$



$c(t) = t\,e^{at}$



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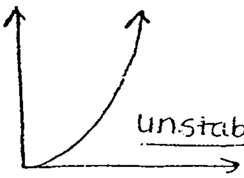
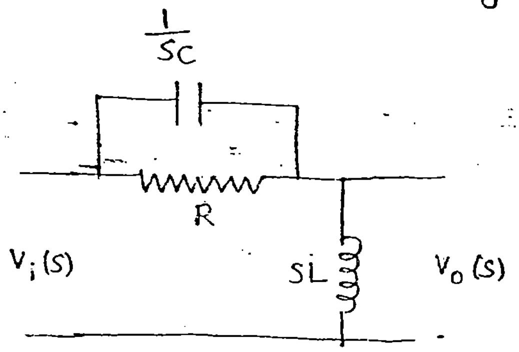
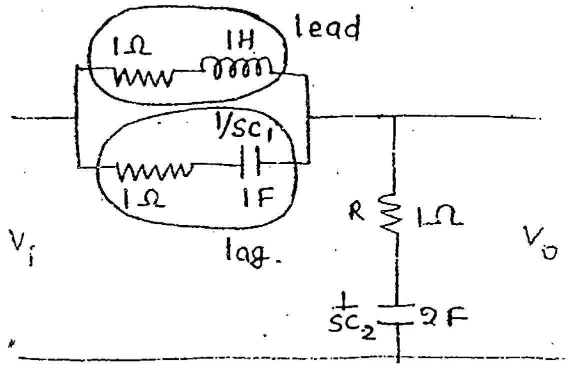
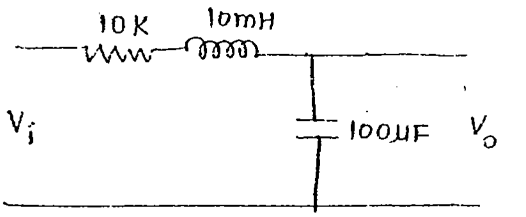


unstable

* Find the T.F. to the following electrical n/w's.







(a)
$$\frac{V_o(S)}{V_i(S)} = \frac{SL}{\dfrac{R \times 1/SC}{R + 1/SC} + SL}$$

$$= \frac{SL\left(R + 1/SC\right)}{\dfrac{R}{SC} + SL\left(R + 1/SC\right)}$$

$$\frac{V_o(S)}{V_i(S)} \Rightarrow \frac{SL\left(SCR+1\right)}{S^2 LCR + SL + R}$$

(b)
$$\frac{V_o(S)}{V_i(S)} = \frac{R + 1/SC_2}{\left[(R+SL)\,||\,(R + 1/SC_1)\right] + \left(R + \frac{1}{SC_2}\right)} = \frac{1 + 1/2S}{\left[(1+S)\,||\,(1 + 1/S)\right]\left[1 + 1/2\right.}$$

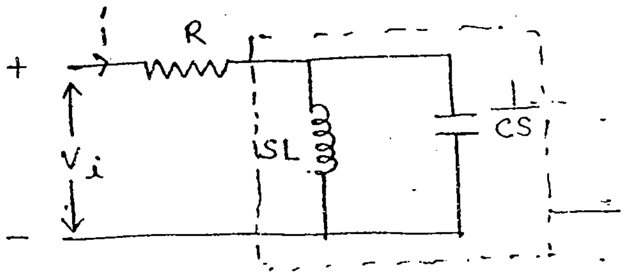
$$= \frac{1 + 1/2S}{\left[\dfrac{(1+S)(1+1/S)}{S+1+1+1/S}\right]} = \frac{2S+1}{2S+2S+1} \times \frac{S+2+1/S}{1+1/S+S+1}$$

$$= \frac{2S+1}{4S+1}$$

(c)
$$\frac{V_o(S)}{V_i(S)} = \frac{1/SC}{R + SL + 1/SC}$$

$$\Rightarrow \frac{1}{RSC + S^2 LC + 1} = \frac{1}{10 \times 100 \times 10^{-6} S + S^2 \cdot 10 \times 10^{-3} \times 10^{-4}}$$

$$= \frac{10^6}{S^2 + 10^6 S + 10^6}.$$



$SL \,||\, \frac{1}{CS}$

$$\frac{SL \times \frac{1}{CS}}{SL + 1/CS}$$

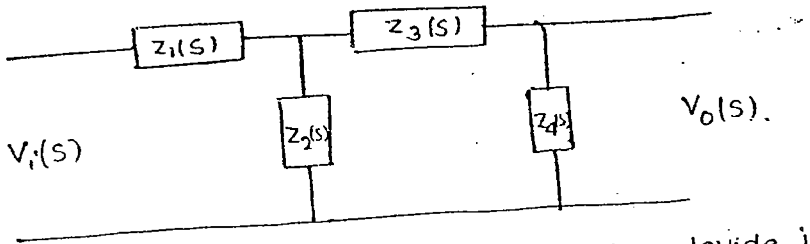
$$= \frac{SL}{S^2LC+1}$$

$$V_i(s) - R\,I(S) - \frac{SL}{S^2LC+1}\, I(S) = 0$$

$$V_I(S) = I(S)\left[R + \frac{SL}{S^2LC+1}\right]$$

$$\frac{I(S)}{V_i(S)} = \frac{S^2LC+1}{S^2LCR + SL + R}$$

Q.

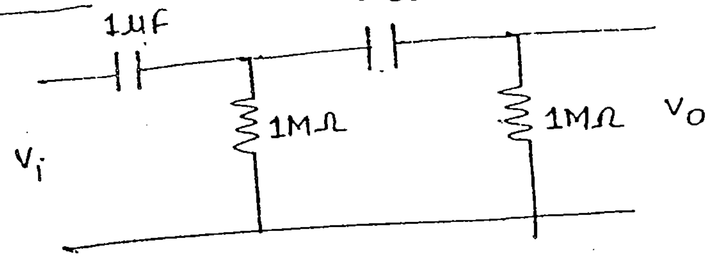


It is an interactive n/w we can't devide it into separate

$$\frac{V_o(S)}{V_i(S)} = \frac{Z_2(S) \cdot Z_4(S)}{Z_1(S)\left[Z_2(S) + Z_3(S) + Z_4(S)\right] + Z_2(S)\left[Z_3(S) + Z_4(S)\right]}$$

$$= \frac{\text{Shunt impedences product}}{1^{st}(\text{Remaining add}) + II^{nd}(\text{Remain add})}$$

Find the T.F.

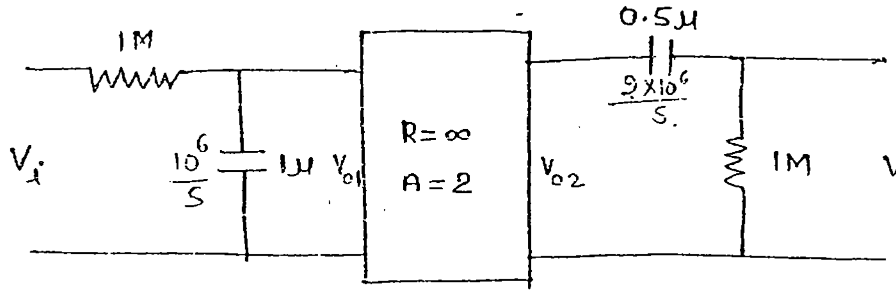


$$\frac{V_o(S)}{V_i(S)} = \frac{\cancel{10^{12}}\ 10^6}{\frac{\cancel{10^6}}{S}\left[10^6 + \frac{2\times10^6}{S} + 10^6\right] + \cancel{10^6}\left[\frac{2\times10^6}{S} + 10^6\right]}$$

$$\Rightarrow \frac{\cancel{10^6}\ S}{\cancel{10^6}\left[\left[2 + \frac{2}{S}\right] + \left(\frac{2}{S} + 1\right)\right]}$$

$$\frac{1}{\frac{4}{s}+2+\frac{4}{s^2}+\frac{2}{s}}$$

$$\frac{s^2}{4s+2s^2+4+2s} = \frac{s^2}{2s^2+6s+4}$$



$$\frac{V_{o1}}{V_i} = \frac{10^6/s}{\frac{10^6}{s}+10^6}$$

$$\Rightarrow \frac{1/s}{1/s+1}$$

$$\Rightarrow \frac{1.s}{s(s+1)}$$

$$= \frac{1}{s+1}$$

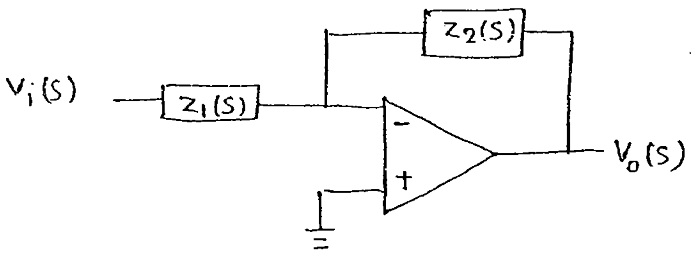
$$\frac{V_o}{V_{o2}} = \frac{10^6}{10^6+\frac{2\times10^6}{s}}$$

$$\Rightarrow \frac{1}{1+2/s}$$

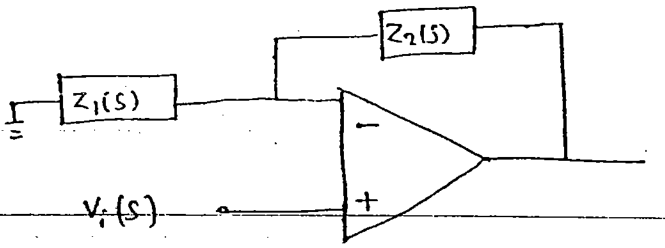
$$= \frac{s}{s+2}$$

$$\frac{V_o}{V_i} = \frac{V_o}{V_{o2}} \cdot \frac{V_{o1}}{V_i} \cdot \frac{V_{o2}}{V_{o1}}$$

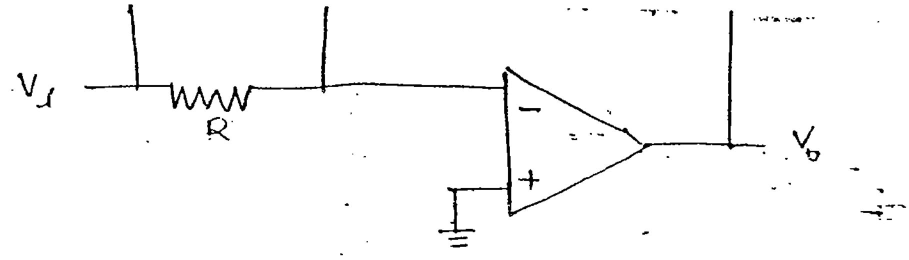
$$= \frac{s}{s+2} \cdot \frac{1}{s+1} (2).$$



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



$$\frac{V_o(s)}{V_i(s)} = \left[1+\frac{Z_2(s)}{Z_1(s)}\right]$$

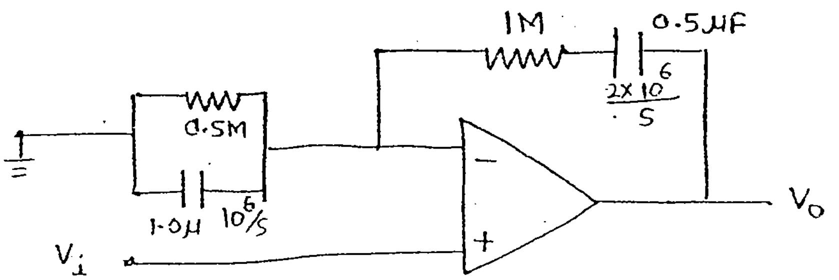


$$\frac{V_o(S)}{V_i(S)} = -\frac{R + 1/CS}{\left(\frac{SR\,L}{R+SL}\right)}$$

$$= -\frac{SCR+1}{CS} \cdot \frac{(R+SL)}{SRL}$$

$$= -\frac{(SL+R)(SCR+1)}{S^2 RLC}$$

Q.



$$\frac{V_o(S)}{V_i(S)} = \left(1 + \frac{\cancel{10^6}\left[1+2/S\right]}{\cancel{10^6}\left[\frac{1/2S}{1/2+1/S}\right]}\right)$$

$$= \left[1 + \frac{S+2}{S} \cancel{\times} \frac{\frac{1}{\cancel{2S}}\times\cancel{2S}}{S+2}\right]$$

$$= \frac{(S+2)^2}{S} + 1.$$

$$\left(10^6 + \frac{2\times10^6}{S}\right)$$

$$\frac{\frac{10^6}{2}\times\frac{10^6}{S}}{\frac{10^6}{2}+\frac{10^6}{S}}$$

$$\frac{\frac{1}{2}\times\frac{10^6}{S}}{(1/2 + 1/S)}$$

* Transfer Function to the differential Equations

write the T.F. to the following System where x is i/p and y is o/p.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 10y = 2\frac{dx}{dt} + x.$$

$$S^n \rightarrow \frac{d^n}{dt^n}$$

$$S^2Y(S) + 5SY(S) + 10Y(S) = 2SX(S) + X(S)$$

$$\frac{Y(S)}{X(S)} = \frac{2S+1}{S^2+5S+10.} = \frac{\text{i/p Related terms}}{\text{o/p Related terms.}}$$

$$\frac{d^3y}{dt^3} + 2\cdot\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 10 = \frac{dx}{dt} + 2x.$$

$$\frac{Y(s)}{X(s)} = \frac{s+2}{s^3+2s^2+5s+10} \rightarrow \text{initial condition.}$$

$$= \frac{s+2}{s(s^3+2s+5)}.$$

write the diff. equation to the given T.F.

$$\frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+5s+6}.$$

$$\left[\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y + \underbrace{(\text{const})}_{0}.\right] = 2\frac{dx}{dt} + 3x.$$

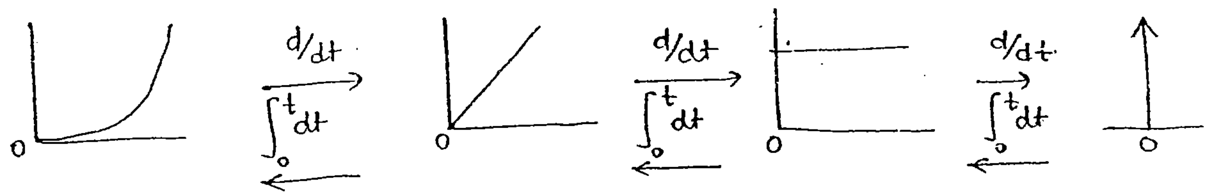
→ T.F. to the signal Response

$$T/F = \left.\frac{L[o/p]}{L[i/p]}\right|_{I_i=0}$$

$$T.F = L[\text{Impulse Response}]_{I_i=0}.$$

unit Ramp Response : (   )

$$T.F. = \left.\frac{L[\text{unit Ramp Response}]}{L[\text{unit Ramp}]}\right|_{I_i=0}$$



the unit step Response of system is
$$y(t) = \left(5/2 - \frac{5}{2}e^{-2t} + 5t\right)$$

its T.F. =?

$$= \left.\frac{\frac{5}{2s} - \frac{5}{2}\cdot\frac{1}{s+2} + \frac{5}{s^2}}{1/s}\right|_{I_i=0}.$$

$$= 5/2 - 5/2\,\frac{s}{s+2} + 5/s.$$

$$\frac{s\cdot 5(s+2) - 5s^2 + 10(s+2)}{} = \frac{10s+10}{}$$

the unit impulse Response of a System is

$$y(t) = -4e^{-t} + 6e^{-2t}$$ its equivalent.

Step Response is

$$= \int_0^t (-4e^{-t} + 6e^{-2t})\, dt$$

$$= \left[+4e^{-t} - \overset{3}{\cancel{6}} \frac{e^{-2t}}{\cancel{2}} \right]_0^t$$

$$= 4e^{-t} - 3e^{-2t} - [4-3] = 4e^{-t} - 3e^{-2t} - 1$$

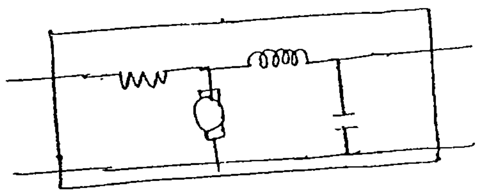
Block - Diagram

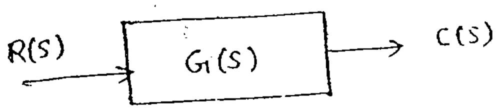
to find the overall T.F. of any practical systems.

the Block diagram is nothing but Short hand pictorial Representation of the system b/w i/p and o/p the systems can be Represented in two ways.

① open loop form.

② closed loop form.

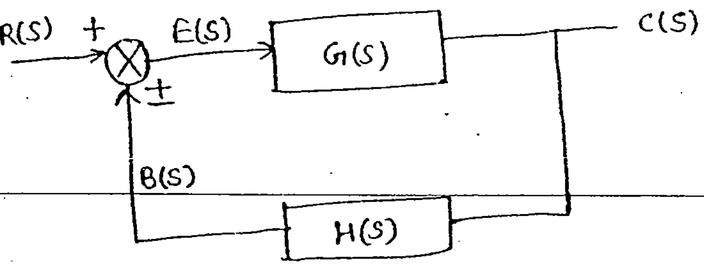




O.L. Form.

$$\text{O.L.T.F.} = \frac{C(S)}{R(S)} = G(S).$$

② Closed Loop form



E(S) → Error Signal

B(S) - Feedback Signal

$$G(S) = \text{Forward path gain} = \frac{C(S)}{E(S)}$$

path gain = $B(S)$

$G(S)\,H(S) \doteq$ Loop gain (OPEN).
↳ OLTF of non unity f/b system.

The Factor $G(S)\,H(S)$ Represent the actual system, the Closed loop system always described in form of OLTF. i.e. $G(S)\,H(S)$.

$H(S) = 1$, $G(S) \longrightarrow$ OLTF of unity f/b system.

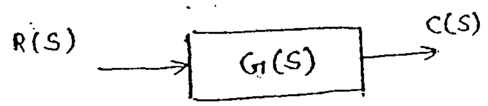
closed loop T.F.

$$\frac{C(S)}{R(S)} = \frac{G(S)\,H(S)}{1 \mp G(S)\,H(S)}.$$

For +ve f/b.
in a practical system phase shift b/w i/p and f/b signal is 0° or $\pm 360^\circ$, whereas For -ve f/b. Phase shift b/w i/p and f/b signal is $\pm 180^\circ$ or out of phase with i/p.

* Comparision b/w open loop and closed loop -

open loop



$$\frac{C(S)}{R(S)} = G(S).$$

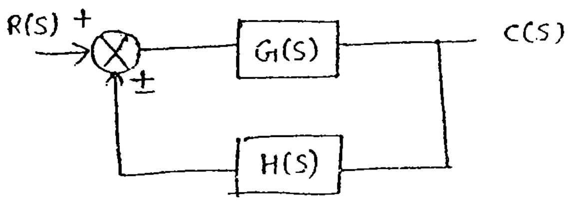
Stability

* in more general open loop system is more stable because there is no factor to affect the system Stability

Accuracy

the OL system Accuracy depends on I/P and process.

closed loop.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 \mp G(S)\,H(S)}$$

the main disadv. of f/b is the gain is decreased by the factor of $[1 + G(S)\,H(S)]$

Stability →
* the closed loop system stability depends on the loop gain. If loop gain = −1 then CL System Stability is affected

$G(S)H(S) = 0 \Rightarrow$ CL Stability = OL Stability.

$G(S)H(S) > 0 \Rightarrow$ CL System becomes more Stable than OL System.

* the CL System Accuracy depends on f/b n/w Ratio. If f/b n/w gives the stable value then the closed loop system become Highly accurate than open loop System.

Sensitivity :-

the open loop System is Highly sensitive for disturbance/ noise/ Envionment conditions because whatever the changes occur in the System the directly affect the o/p.

the closed loop Control System is less sensitive for disturbance/ noise/ E.C. because change in o/p is very less. [less than 1%] which is called Improving Sensitivity.

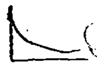
Bandwidth. (open Loop & closed loop).

$\boxed{\text{For any practical system, the gain} \cdot \text{BW product is Constant}}$

with F/b. the gain is decreased by the factor of $1+G(s)H(s)$ that means BW must be increased by Factor of $1+G(s)H(s)$

the BW Represents the speed of the Response
the Lage BW gives the very quick response

$\boxed{\uparrow BW \propto \frac{1}{t_r \downarrow}}$ $\boxed{BW = \frac{0.35}{t_r}}$

with f/b – Response decays Fastly. 

Reliability

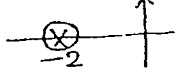
Reliability of the system completely depends on no. of discrete component

the open loop system having less no. of component hence It is more Reliable.

* in open loop control System, o/p is not measured, no errors are generated, Sensors are not Required

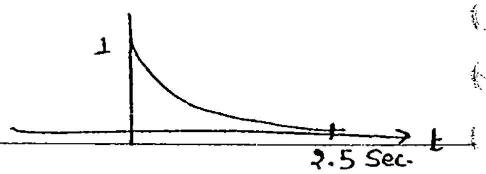
the o/p must be measured, the errors are generated, Sensors must be required.

w/o F/B. $G(s) = \frac{1}{s+2}$

Reference


$\tau = 0.5$ sec.
BW = 2 Hz.

System Response $= 1 \cdot e^{-2t}$

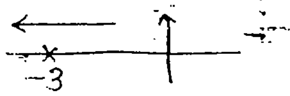


with −ve F/b.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

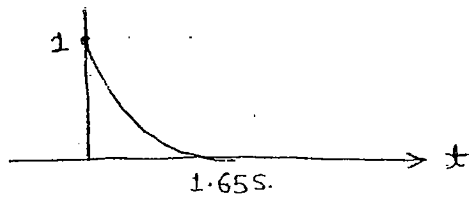
Relative Stability improved.

$$\frac{C(s)}{R(s)} = \frac{1}{s+3}.$$



$\tau = 0.33$ sec ⇊

System Response $= 1 \cdot e^{-3t}$



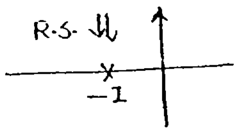
BW = 3Hz ↑↑

Very quick Response

with +ve F/b.

$$\frac{C(s)}{R(s)} = \frac{G}{1-G}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s+1}$$

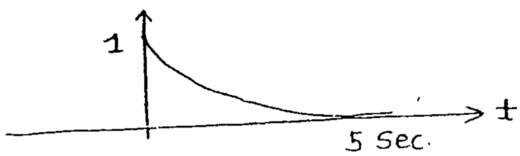


$\tau = 1$ sec ↑↑

BW = 1 Hz.

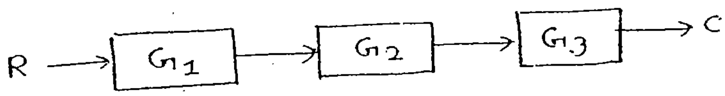
Slow Response.

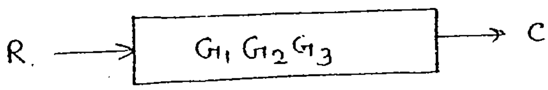
System Res. $= 1 \cdot e^{-t}$



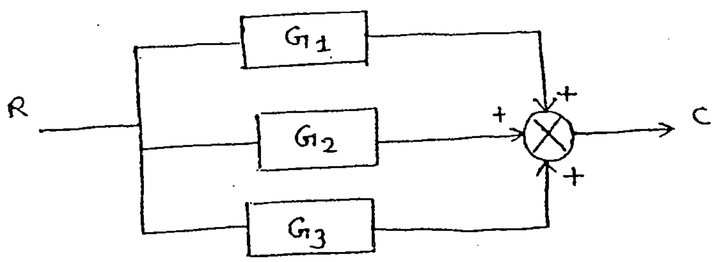
Block Diagram:- Reduction techniques.

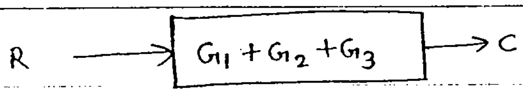
① Blocks are in Series / cascade.



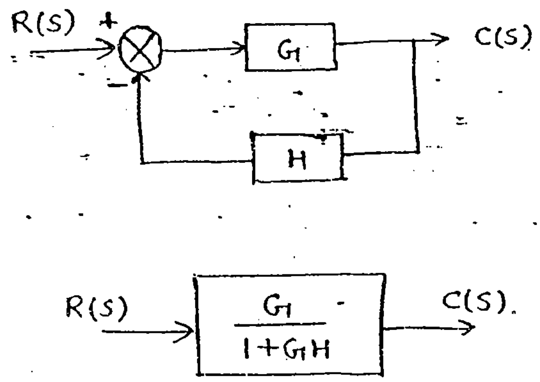


② Blocks are in parallel.

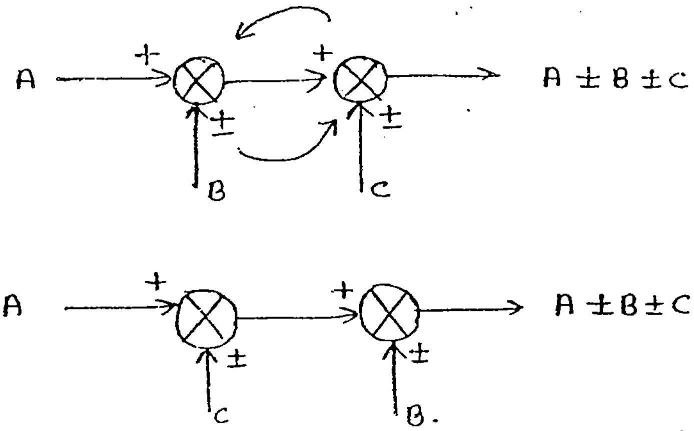




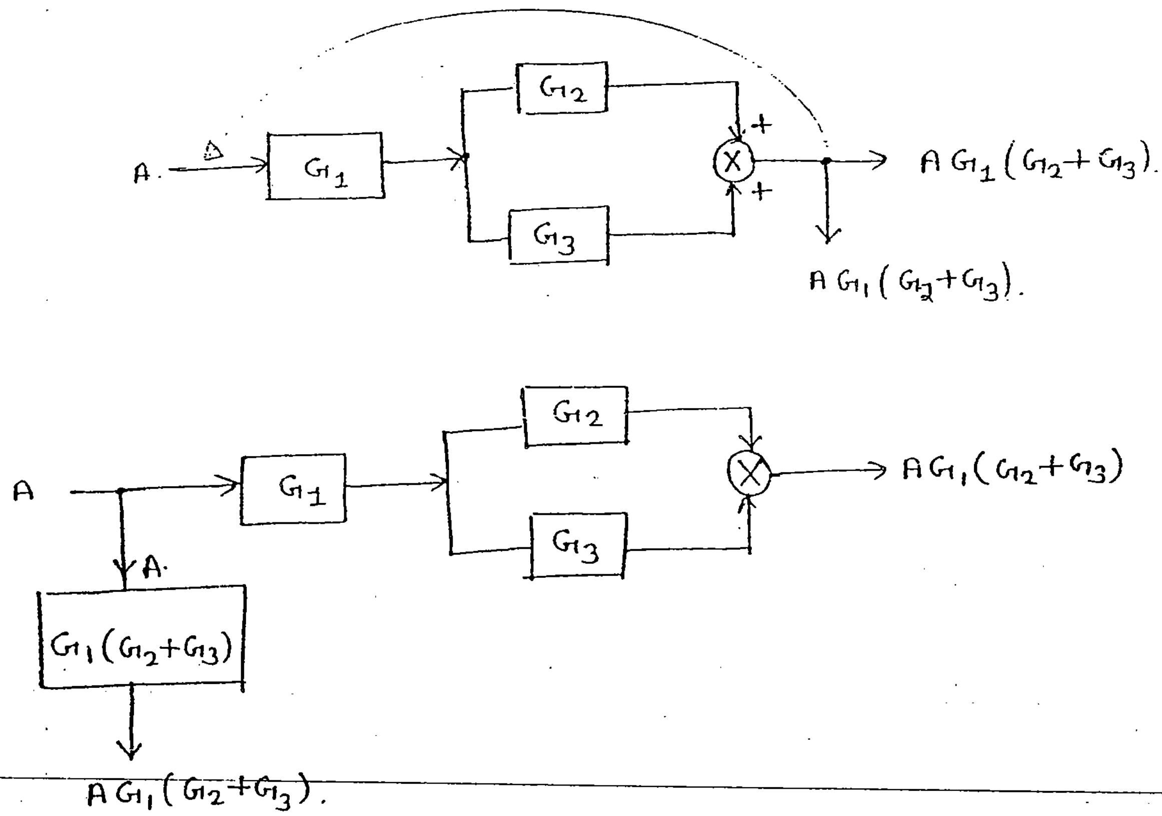
③ Loop.



④ Interchanging the summing point −

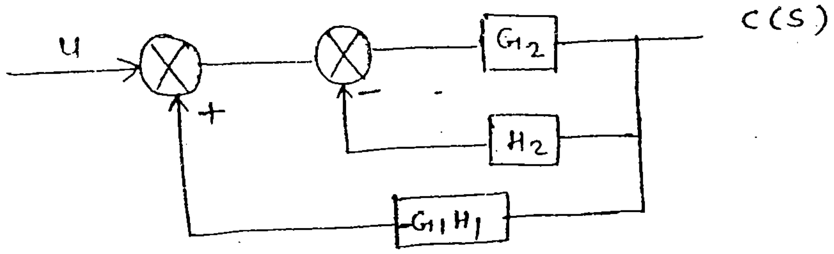


⑤ Adjusting the Block gain and take-off Point −



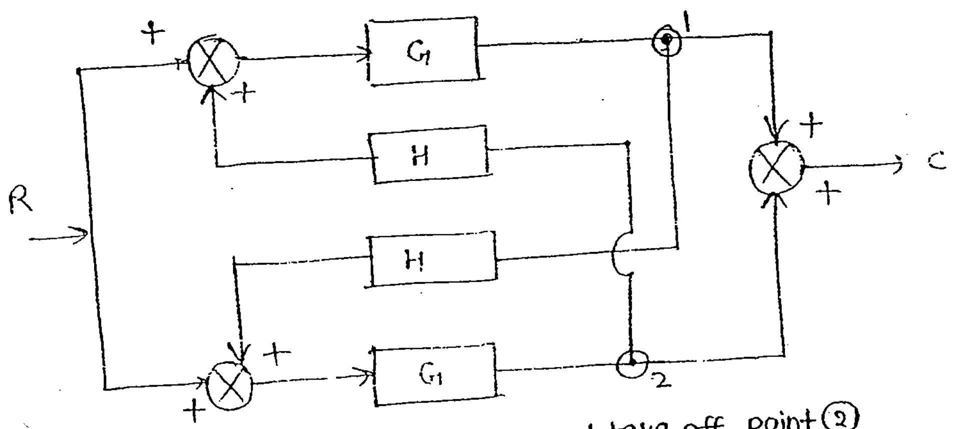
$R=0$

$$\left.\frac{C}{U}\right|_{R=0} = \frac{G_2}{1+G_2H_2 + G_1G_2H_1}$$

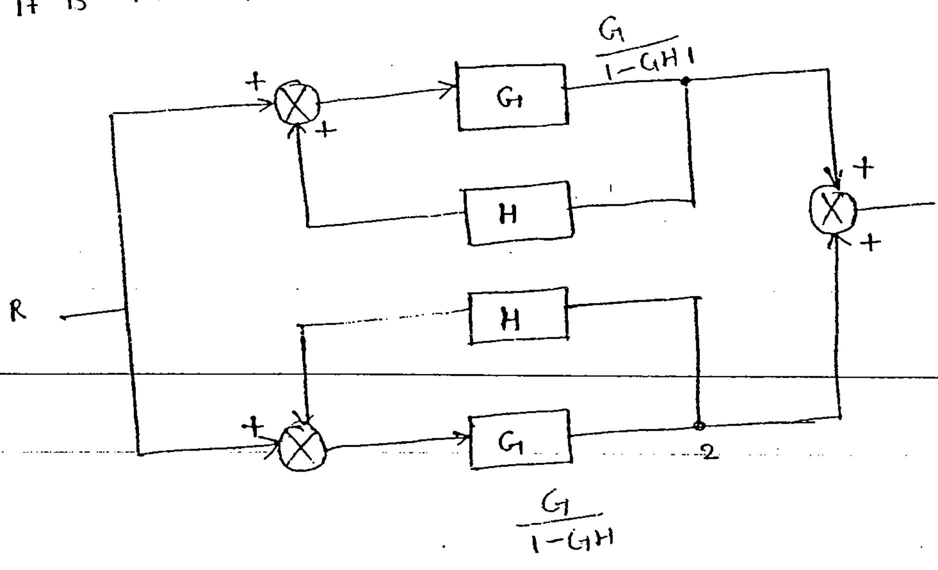


$$C = \frac{G_1G_2R + G_2U}{1+G_2H_2+G_1G_2H_1}$$

(*) Find the gain of system given below.



(*) In the above system, gain at take off point ① = gain at take off point ② Hence we can interchange take off point ① & ② If it is not equal we slove it by SFG.



$$C = \frac{2G}{1-GH}$$

Q.
The impulse response of a unity f/b. Control System given by

$$c(t) = (-te^{-t} + 2e^{-t})$$

its equivalent open loop T.F. is..

$$C(S) = \frac{-1}{(S+1)^2} + \frac{2}{S+1}, \quad R(S) = 1.$$

$$CLTF = L\left[\text{Impulse Response}\right]_{I_1=0}.$$

$$\frac{C(S)}{R(S)} = \frac{-1+2S+2}{(S+1)^2} = \frac{2S+1}{(S+1)^2}.$$

$$\frac{G(S)}{1+G(S)} = \frac{2S+1}{(S+1)^2}$$

$$\frac{1+G(S)}{G(S)} = \frac{S^2+2S+1}{2S+1}$$

$$\frac{1}{G(S)} + \cancel{1} = \frac{S^2}{2S+1} + \cancel{1}$$

$$\boxed{G(S) = \frac{2S+1}{S^2}}$$

$$\frac{G}{1+G-G} = G = \frac{2S+1}{S^2+2S+1-2S-1} = \frac{2S+1}{S^2}$$

to get the O.L.T.F. From closed loop Subtract the numerator term in the denominator when the F/b is unity.

to get the C.L.T.F. from open loop add the numerator term in the denominator when the F/b is unity

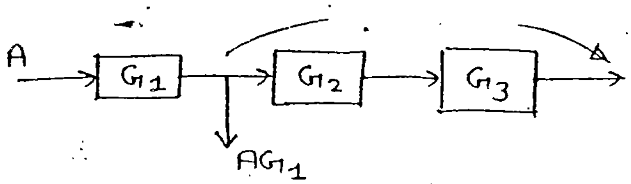
Find the open loop DC gain of a unity F/b System with CLTF

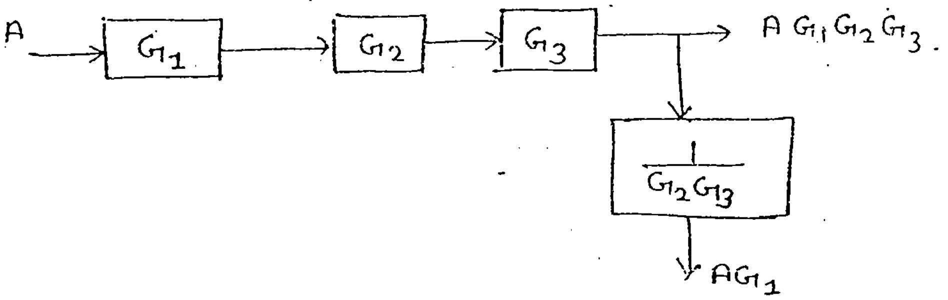
$$\frac{C(S)}{R(S)} = \frac{2S+4}{S^2+5S+13}.$$

DC gain $f = 0$, $S = j\omega = j2\pi f = 0$.

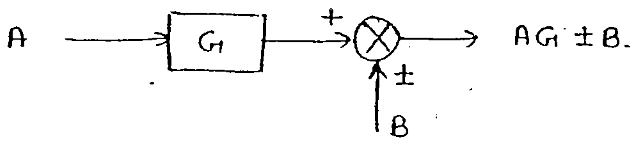
CL DC gain $\frac{C(S)}{R(S)} = \frac{0+4}{0+0+13} = 4/13.$

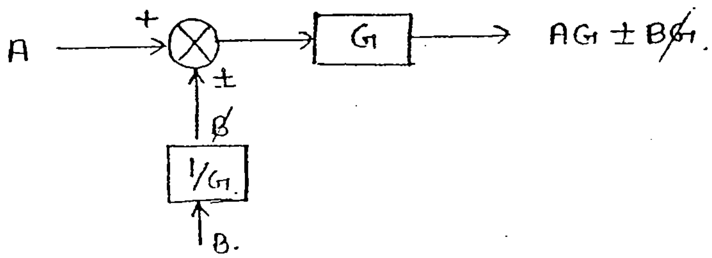
$$G(S) = \frac{\cancel{2S}^{0}+4}{\cancel{S^2}_{0}+\cancel{3S}_{0}+9} = 4/9$$



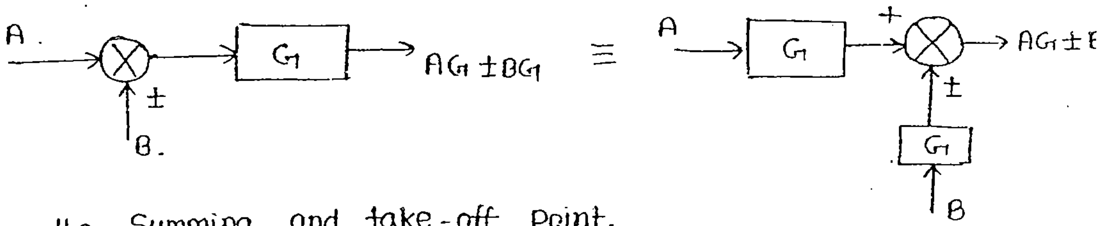


⊛ Adjusting the Block gain and Summing point.

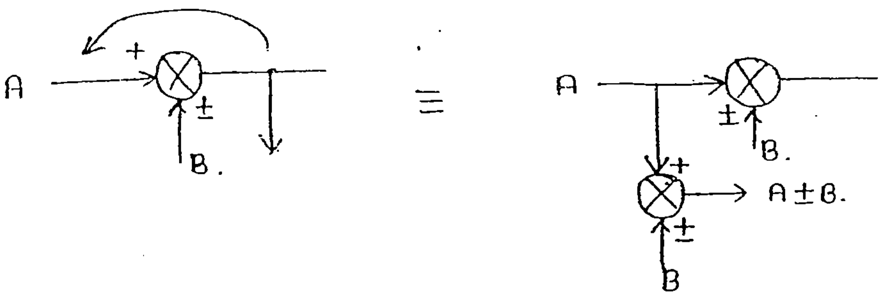




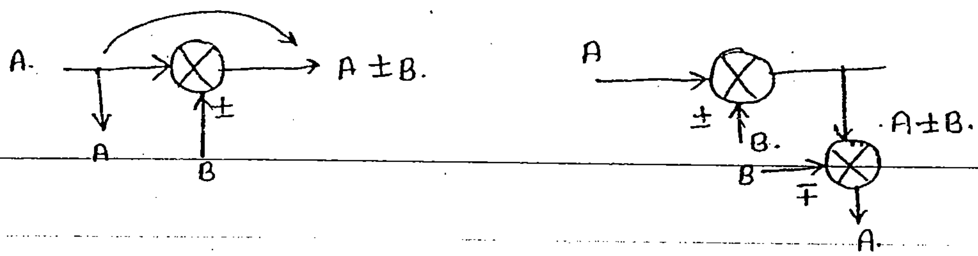
<u>other way</u>



⊛ Adjusting the Summing and take-off point.

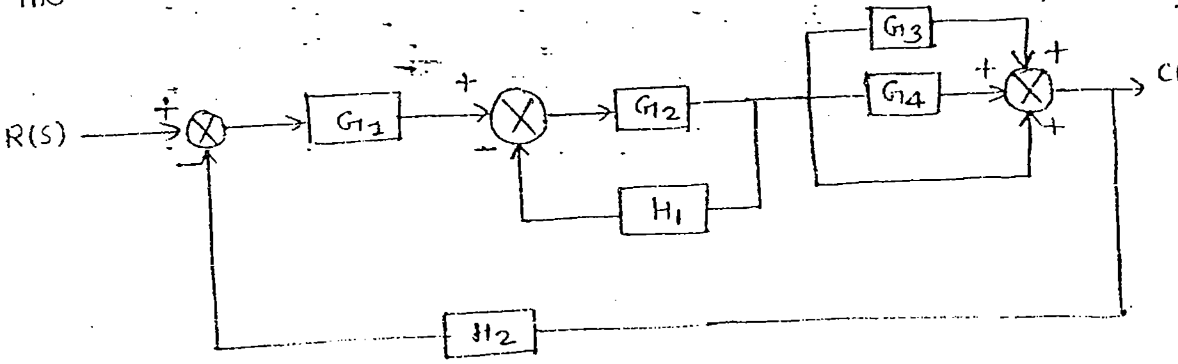


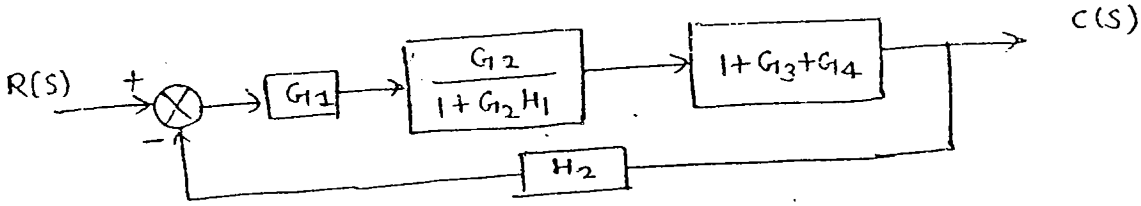
<u>other way</u>

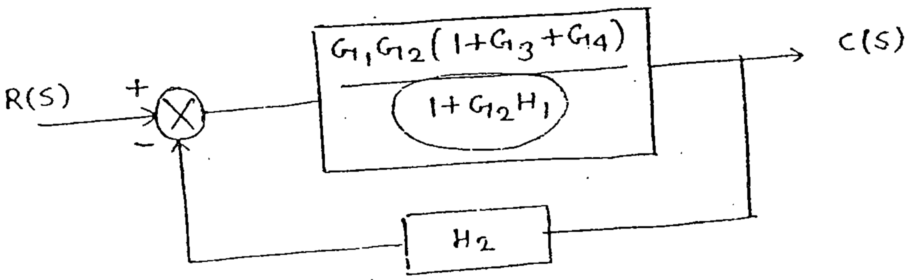


Problem

Find the overall T.F. of the given system.



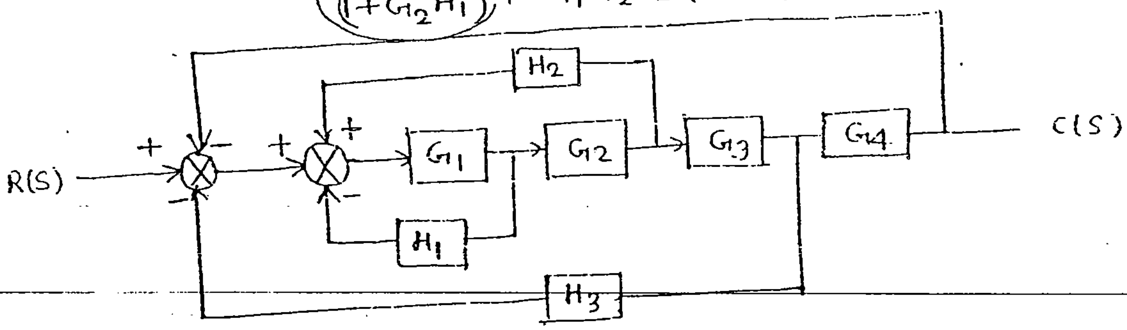




$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1G_2(1+G_3+G_4)}{1+G_2H_1}}{1+\dfrac{G_1G_2(1+G_3+G_4)H_2}{(1+G_2H_1)}}$$

$$= \frac{G_1G_2(1+G_3+G_4)}{(1+G_2H_1)+G_1G_2H_2(1+G_3+G_4)}$$

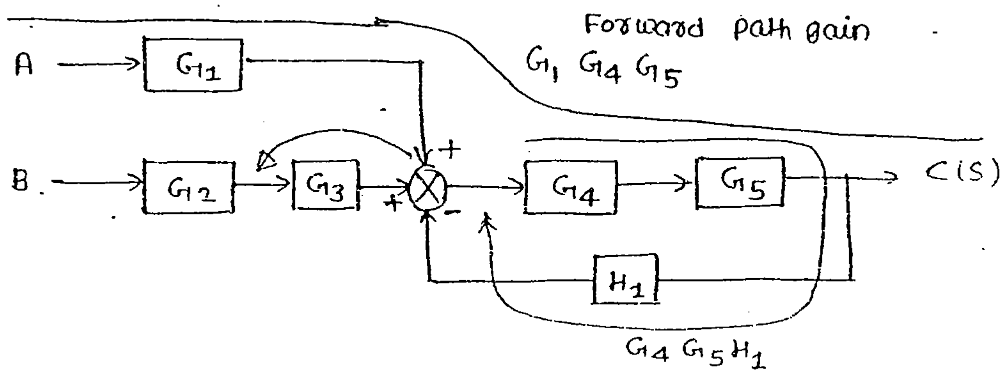
Q.

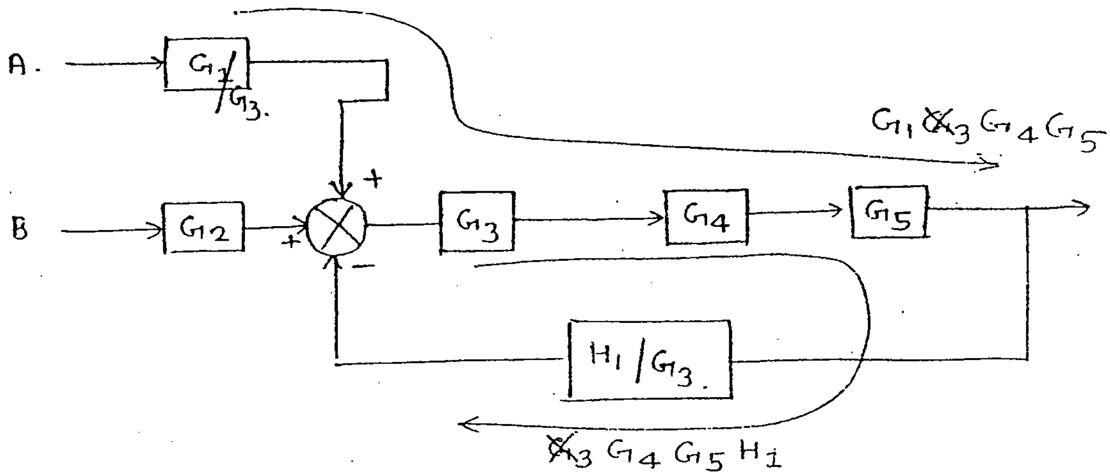


$$\frac{\dfrac{G_1G_2}{1+G_1H_1}}{1+G_1H_1}$$

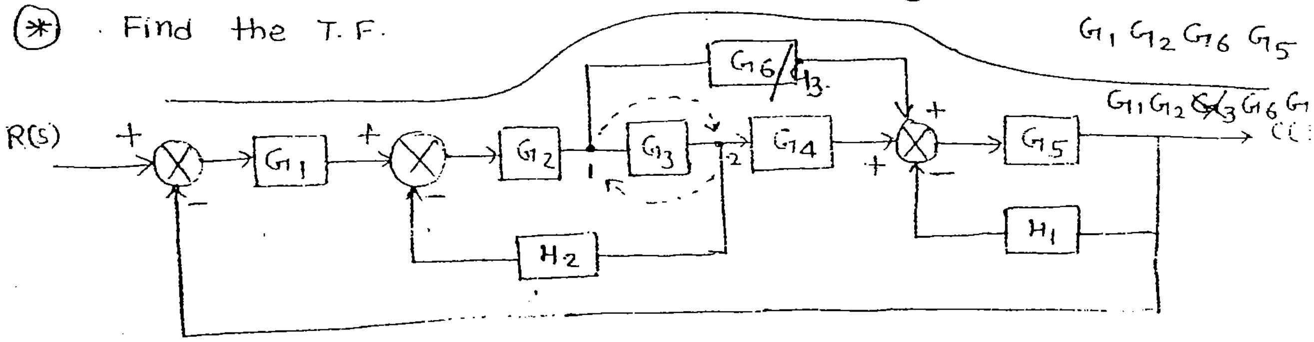
$$\frac{G_1\, G_2\, G_3\, G_4}{1+G_1H_1 - G_1G_2H_2 + G_1G_2G_3H_3 + G_1G_2G_3G_4}$$

(*) Draw the equivalent B.D. to the given.

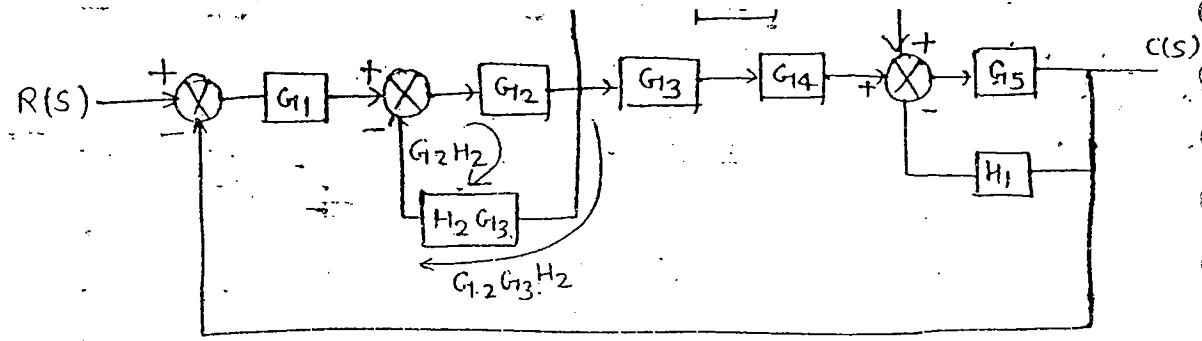




(*) Find the T.F.

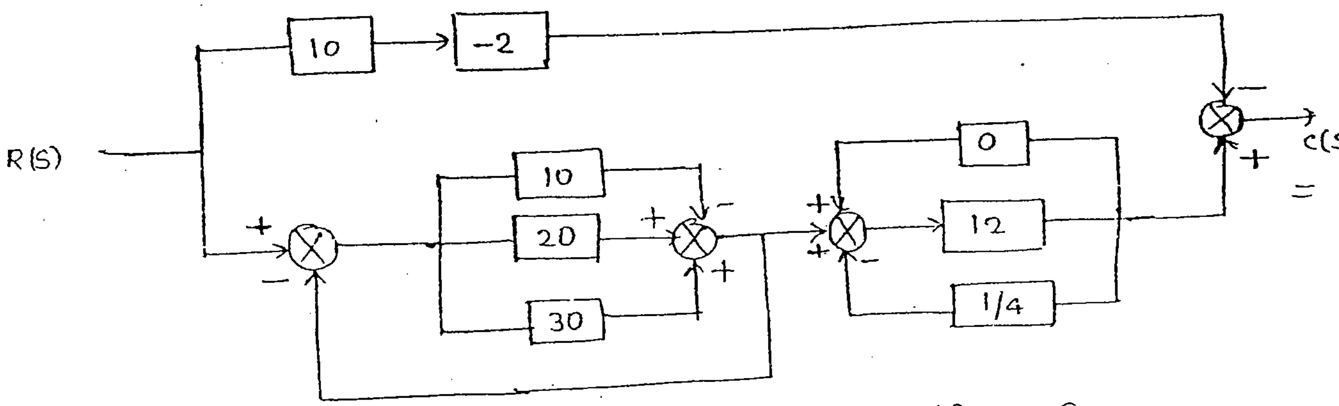


changes are only additional forward path/ Feedback path.



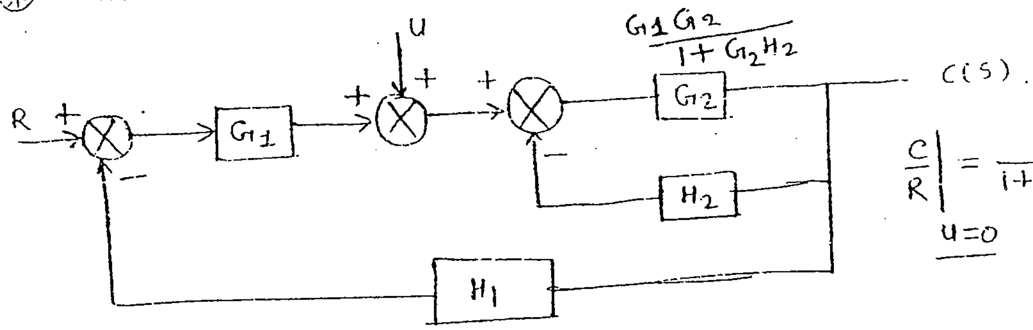
$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 (G_3 G_4 + G_6) G_5}{(1+G_2 G_3 H_2)(1+G_5 H_1) + G_1 G_2 (G_3 G_4 + G_6) G_5}$$

(*) Find the gain of the system given below.



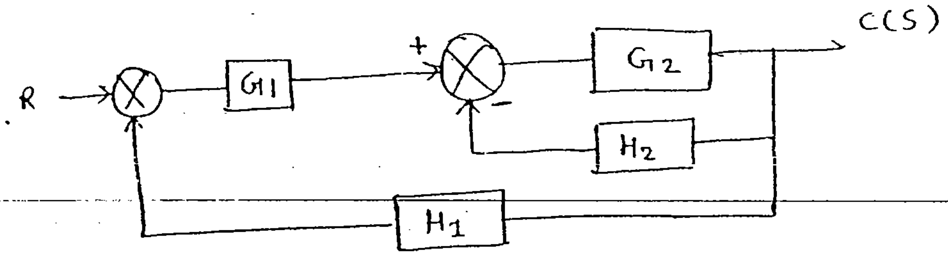
$$\frac{40}{1+40} \cong \left(\frac{40}{41}\right) \approx 1 \qquad \frac{12}{1+3} = 3$$

(*) Find the o/p due to i/p R, and u. acting Simultaneously



$$\left.\frac{C}{R}\right|_{u=0} = \frac{G_1 G_2}{1+G_2 H_2 + G_1 G_2 H_1}$$

u=0



* Response of te^{-5t} the i/p must be equal to.

→ the given System is openloop because no where f/b is ment

$$g(t) = 5e^{-5t}$$
$$c(t) = te^{-5t}$$
$$r(t) = ?$$

$$\frac{C(S)}{R(S)} = G(S)$$

$$R(S) = \frac{C(S)}{G(S)} = \frac{1}{(S+5)^2} \Big/ \left(\frac{5}{S+5}\right)$$
$$= \frac{1}{5(S+5)}$$

$$\xrightarrow{\text{I.L.T.}} \boxed{r(t) = \tfrac{1}{5}e^{-5t}}$$

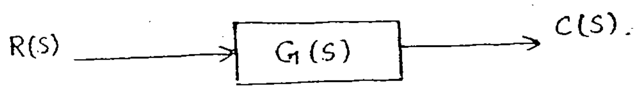
** Signal Flow graphs →

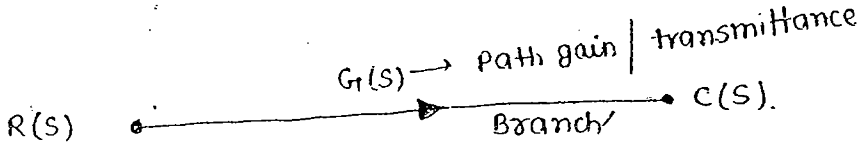
It is a graphical Representation of, a set of Linear algebraic equation b/w Input and O/P,

The set of linear algebraic equations Represents the systems.

* The Signal Flow graphs are Basically developed to avoid the mathematical calculation, The SFG Analysis is very easy compare to Solving Integrodifferential equⁿ or Linear algebraic eq.

* Construction of SFG to the Linear algebraic equation -





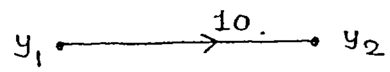
* SFG and B.D. are unidirectional

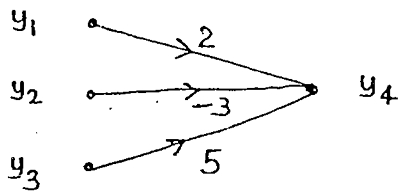
o/p node, Path gain, Input node

① $y_2 = 10 y_1$

② $y_4 = 2y_1 - 3y_2 + 5y_3$.

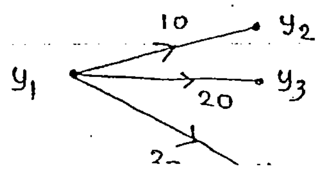
③ $y_2 = 10y_1$
$y_3 = 20y_1$
$y_4 = 30y_1$

①


②


(addition Route)

(many to one)

③


(one to many

(transm Rule)

The nodes in SFG is nothing but variables i.e. current and voltage.

The path gains are nothing but impedences or admittances.

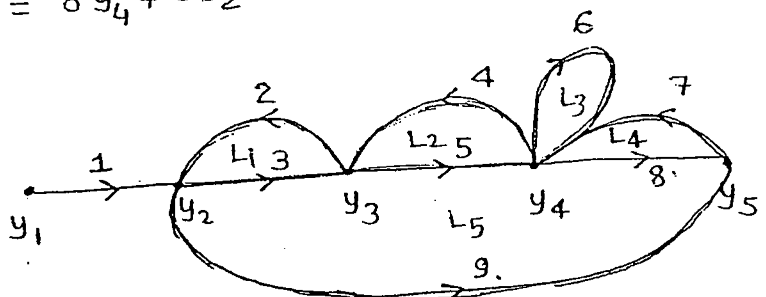
Construct the SFG to the following set of linear algebraic equation

$$y_2 = y_1 + 2y_3$$

$$y_3 = 3y_2 + 4y_4$$

$$y_4 = 5y_3 + 6y_4 + 7y_5$$

$$y_5 = 8y_4 + 9y_2$$



Find the no. of Forward path, no. of Individual loops, no. of two nontouching loop to the above SFG.

Forward path →

It is a path from i/p to o/p

Loop →

Loop is a path which terminates at same node where it is started.

Non-touching loop

If there is no common node b/w two or more loops then it is called non-touching loops.

i/p Node

The node it has only outgoing branches.

o/p Node

The node it has only incoming branches.

Chain node / Link node –

The node which have both incoming and outgoing branches.

The condition to select correct forward path or loop is each node should be ~~touches~~ only one

Non-touching loop -

$L_1 L_2$ ✗ $L_2 L_3$ ✗ $L_3 L_4$ ✗ $L_4 L_5$ ✗

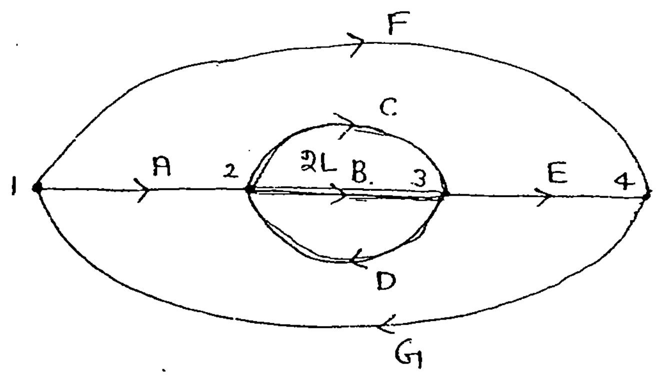
L_3 ✓ L_4 ✗ L_5 ✗

L_4 ✓ L_5 ✗

L_5 ✗

2 Non touching loop.

Q.



No. of forward path - ABE, F, ACE. (3).

No. of Loops. - 5.

No. of two Nontouching loop - 2.

$D \longrightarrow 2L$
- L_1 BD $\longrightarrow$ 2 3
- L_2 CD $\longrightarrow$ 2 3

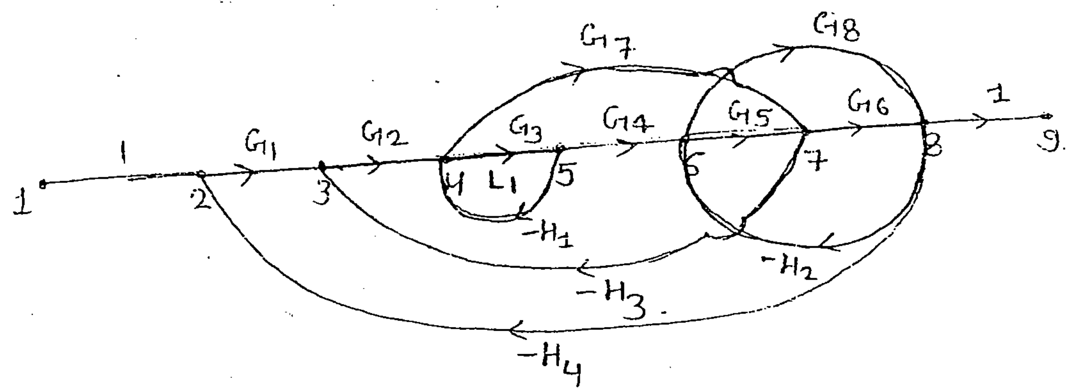
$G \longrightarrow$
- L_3 — ABEG — 1 2 3 4
- L_4 — ACEG — 1 2 3 4
- L_5 — FG — 1 4

$\left.\begin{array}{l} L_1 L_5 \\ L_2 L_5 \end{array}\right\} \underline{\underline{2}}$

(Here L_1 have (2, 3) - L_5 (1, 4))

So $\underline{L_1 L_5}$

Same no. is not there then It called non. touching.



$H_1 \rightarrow L_1 \rightarrow 4, 5$

two Forward path)

H_2 (2L)
- $L_2 \rightarrow 6, 7, 8$
- $L_3 \rightarrow 6, 8$

H_3 (2L)
- L_4 — 3, 4, 5, 6, 7
- L_5 — 3, 4, 7

H_4 (3L)
- L_6 — 2, 3, 4, 5, 6, 7, 8
- L_7 — 2, 3, 4, 7, 8
- L_8 — 2, 3, 4, 5, 6, 8

8 LOOPS

two Non-touching Loops.

$L_1 L_2$ (4, 5, 6, 7, 8)

$L_1 L_3$ (4, 5, 6, 8)

$L_3 L_5$ (6, 8, 3, 4, 7).

three non-touching
select the pair of two nontouching Loop.

Massons gain Formula –

Purpose – * Find the ratio of any two nodes.

* to Find the overall T.F.

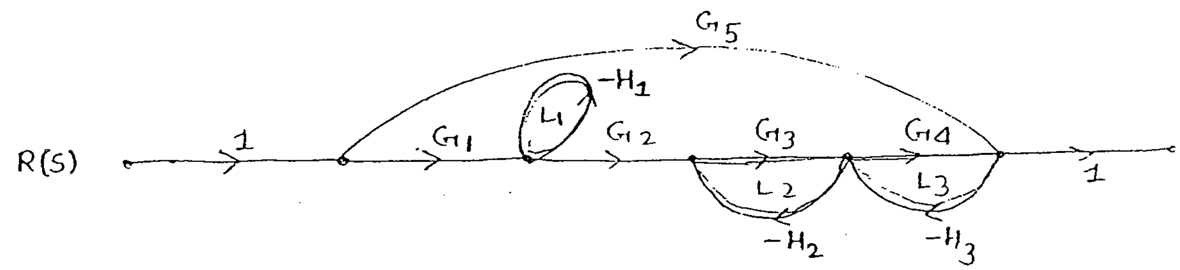
$$\text{overall T.F.} = \frac{\sum_{K=1}^{i} P_K \Delta_K}{\Delta}$$

$P_K \rightarrow K^{th}$ Forward path gain.

$\Delta \rightarrow 1 - \sum$ (Individual loop gain) + (Sum of gain products of two non touching loop) – $\sum$ (gain products of three non-touching loop) + $\sum$ (gain products of four non-touching loop)

Note → In Δ take opposite sign for odd no. of non-touching loop and take same sign for even no. of non-touci loop.

$\Delta_K \rightarrow$ It is obtained from Δ by removing the loops touching the K^{th} Forward path.



Forward path → $P_1 \rightarrow G_1 G_2 G_3 G_4$

→ $P_2 \rightarrow G_5$

Loops → $\underset{L_3}{-G_4 H_3}$, $\underset{L_2}{-G_3 H_2}$, $\underset{L_1}{-H_1}$

$$\boxed{T.F. = \frac{1 \cdot (G_1 G_2 G_3 G_4) + G_5 \begin{bmatrix} 1 + H_1 + G_3 H_2 \\ + G_3 H_1 H_2 \end{bmatrix}}{1 + H_1 + G_3 H_2 + G_4 H_3 + G_3 H_1 H_2 + G_4 H_1 H_3}}$$

Nontouching loop

$L_1 L_2$ $L_1 L_3$

$\Delta = 1 - (L_1 + L$

$+(L_1 L_2 + l$

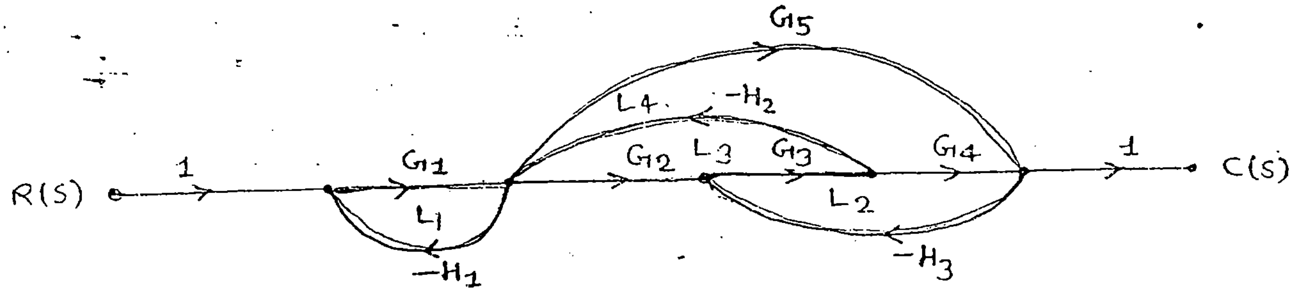
Δ_1 – to the first Forward pc all loops a touching

$\Delta_1 = 1 - (0 + 0 + 0$

$\Delta_1 = 1$

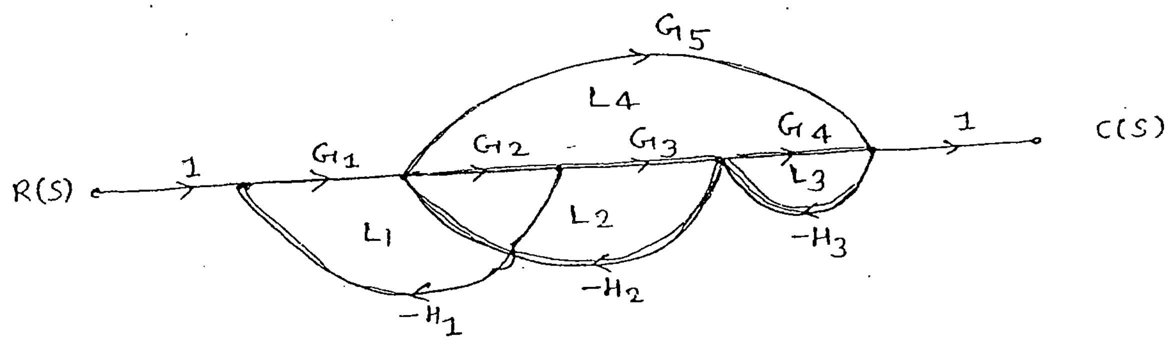
$\Delta_2 = 1 - (L_1 + L_2$

$+ L_1 L_2$



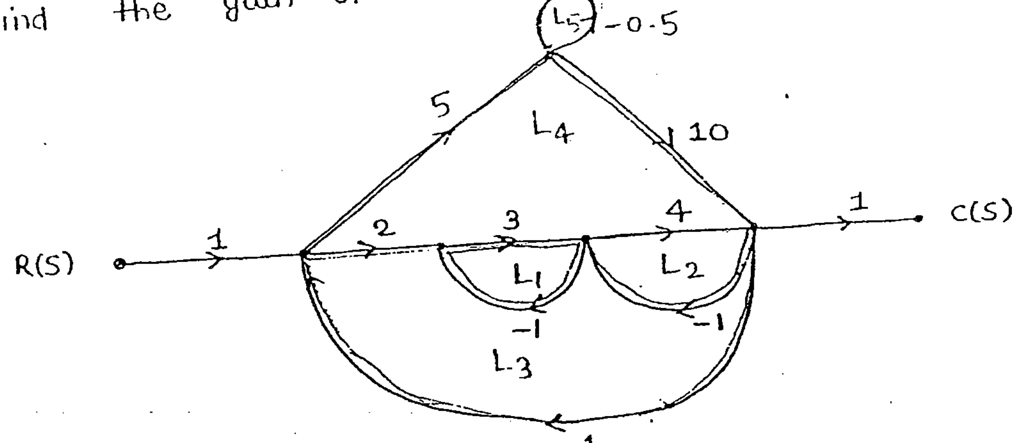
$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 H_1 + G_2 G_3 H_2 + G_3 G_4 H_3 - G_5 H_3 G_3 H_2 + \underbrace{G_1 H_1 G_3 G_4 H_3}_{L_1 L_2}}$$

Q.



$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 H_1 + G_2 G_3 H_2 + G_4 H_3 + G_5 H_3 H_2 + G_1 H_1 G_4 H_3 G_2}$$

* Find the gain of the system given below.

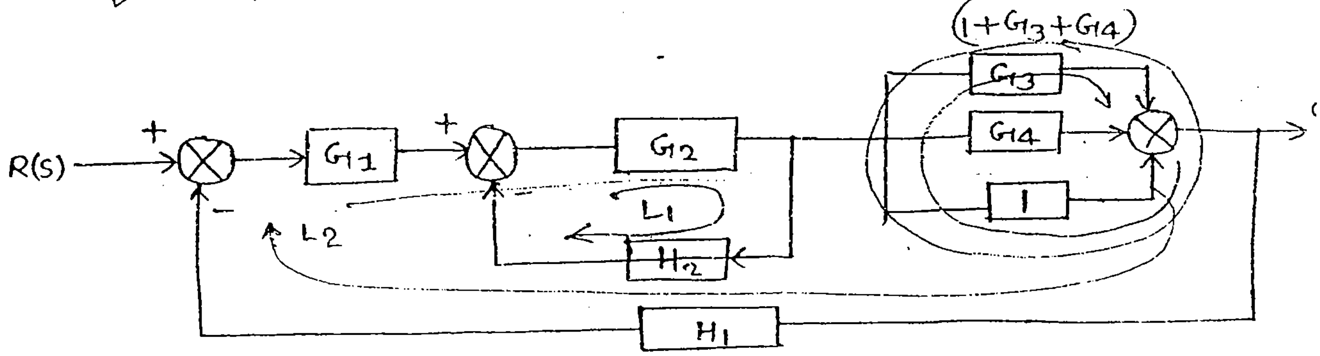


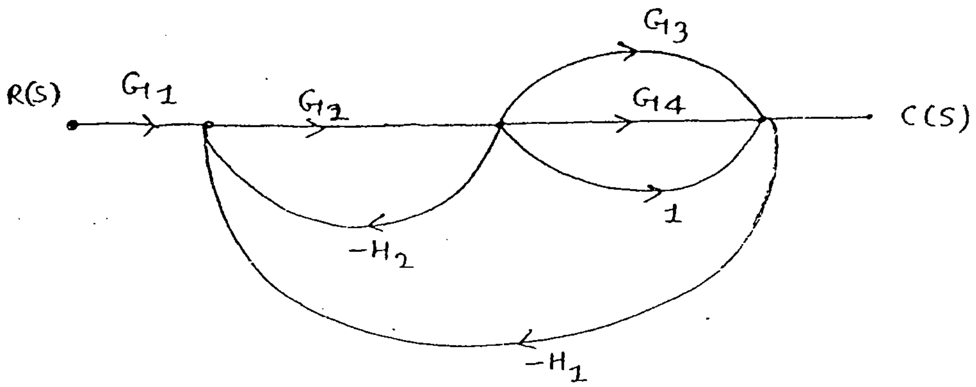
$$\frac{C(S)}{R(S)} = \frac{24(1+0.5) + 25(1+3+4+24) \times 2}{1+3+4+24+0.5+(1.5)+2+12}$$

$$\frac{C(S)}{R(S)} = \frac{24(1+0.5)+50(1+3)}{1+3+4+24+50+0.5+\underbrace{3\times50}_{L_1L_4}+\underbrace{3\times0.5}_{L_1L_5}+\underbrace{4\times0.5}_{L_2L_5}+\underbrace{24\times\cdot}_{L_3L}}$$

$$= 0.95.$$

→ Find the transfer function.

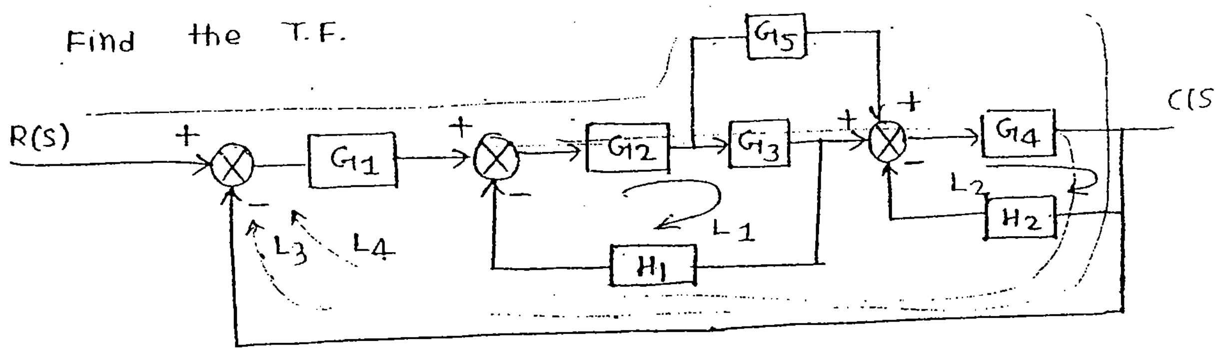




$$\frac{C(S)}{R(S)} = \frac{G_1G_2(1+G_3+G_4)}{1+G_2H_2+G_1G_2(1+G_3+G_4)H_1}$$

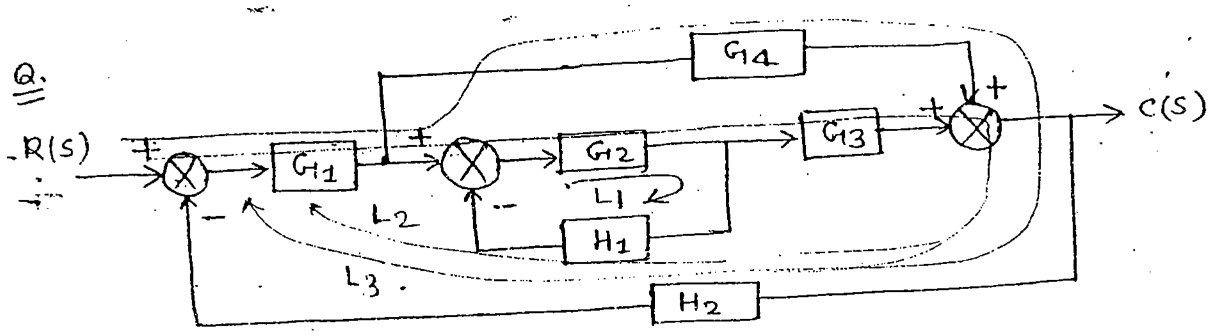
→ Find the T.F.

Q.



$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4+G_1G_2G_5G_4}{1+G_2G_3H_1+G_4H_2+G_1G_2G_3G_4+G_1G_2G_5G_4+G_2G_3H_1G_4H}$$

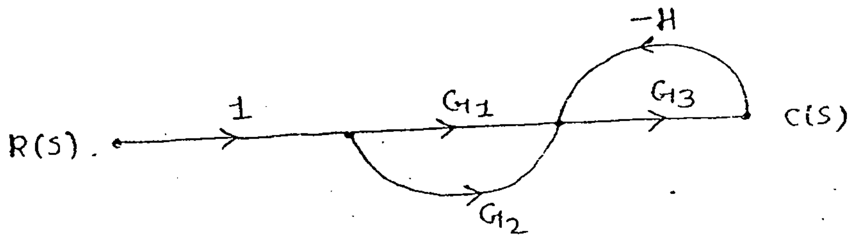
Q.



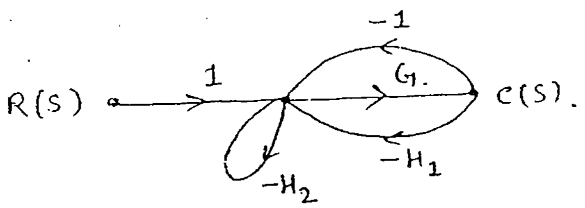
$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3 + G_4(1 + G_2 H_1)}{1 + G_2 H_1 + G_1 G_2 G_3 H_2 + G_4 H_2 + G_2 H_1 G_4 H_2}.$$

Q. Find the T.F.

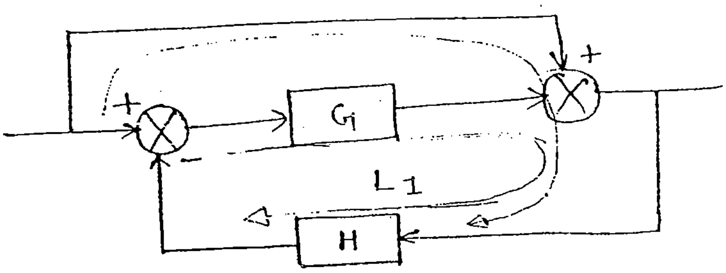
(I)



(II)



(III)



(one loop)

① $C/R = \dfrac{(G_1 + G_2)\, G_3}{1 + G_3 H}$

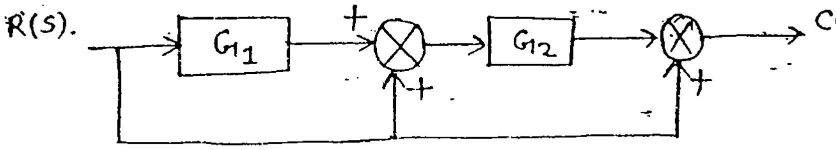
② $C/R = \dfrac{G_1}{1 + G_1 + G_1 H_1 + H_2}$

③ $C/R = \dfrac{G_1 + 1}{1 + G_1 H}.$

Find the T.F.

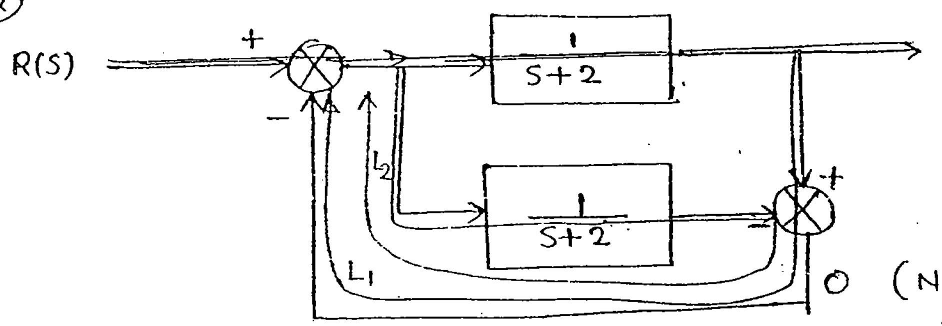
Linear system = interactive system.

(1)



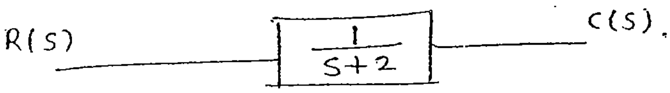
$\div\ (G_1G_2+G_2+1)$

(2)

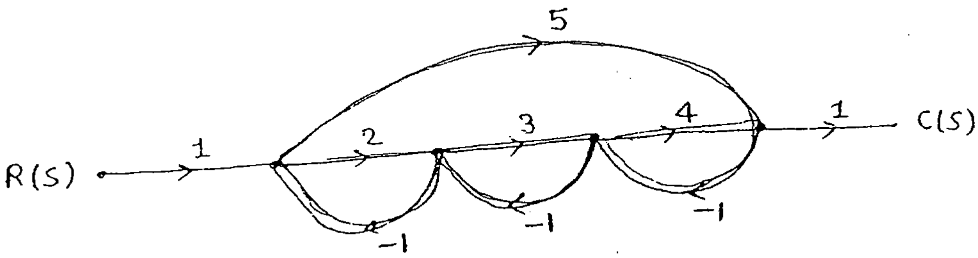


O (No Feedback)

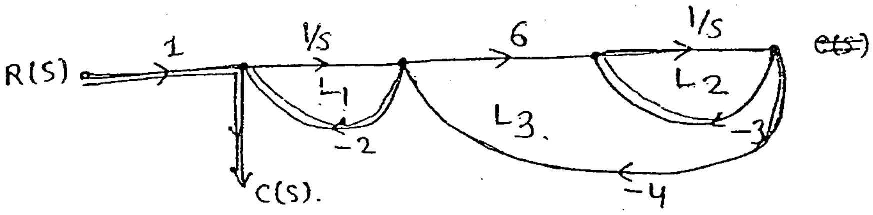
$$\frac{C(S)}{R(S)} = \frac{\frac{1}{S+2}}{1+\frac{1}{S+2}-\frac{1}{S+2}} = \frac{1}{S+2}$$



(3)



(4)



Ans.

(3) → $$\frac{C(S)}{R(S)} = \frac{24+5(1+3)}{1+2+3+4+8+5} = \frac{44}{23}.$$

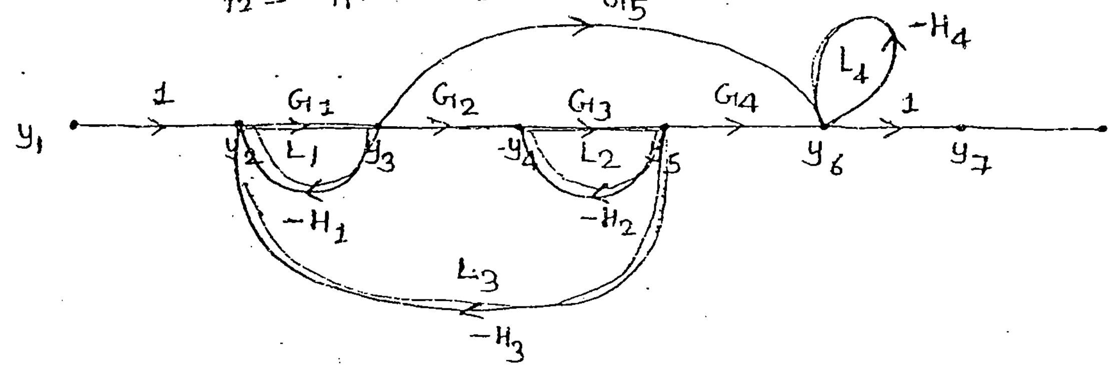
(4) − $$\frac{C(S)}{R(S)} = \frac{(\cancel{6/s^2})\ 1\cdot(1+3/s+24/s)}{1+2/s+3/s+24/s+6/s^2}$$

$$= \frac{s(s+27)}{s^2+29s+6}$$

(*) Find $\frac{Y_6}{Y_1}$; $\frac{Y_2}{Y_1}$, $\frac{Y_5}{Y_1}$; ------

Ratio of any two nodes.

$$Y_2 = Y_1 \times 1 - Y_3 H_1 - Y_5 H_3.$$



$$\frac{Y_7}{Y_1} = \frac{Y_6}{Y_1} = \frac{G_1G_2G_3G_4(1) + G_1G_5(1+G_3H_2)}{1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_3 + G_1G_3H_1H_2 + \underset{L_4}{H_4} + G_1H_1H_4 + G_3H_2H_4 + G_1G_2G_3H_3H_4 + \underline{G_1H_1G_3H_2H_4}}$$

(3 non touching)

$$Y_7 = Y_6 \times 1 = Y_6$$

(By write node eq. we should consider only incoming Branch)

$$* \quad \frac{Y_5}{Y_1} = \frac{G_1G_2G_3(1+H_4)}{\Delta}$$

$$* \quad \frac{Y_2}{Y_1} = \frac{1(1+G_3H_2+H_4+G_3H_2H_4)}{\Delta}$$

* * *

$$\boxed{\frac{Y_7}{Y_2} = \frac{Y_7/Y_1}{Y_2/Y_1}}$$

$$* \quad \frac{Y_7}{Y_2} = \frac{G_1G_2G_3G_4 + G_1G_5(1+G_3H_2)}{\cancel{1}\cdot(1+G_3H_2+H_4+G_3H_2H_4)}$$

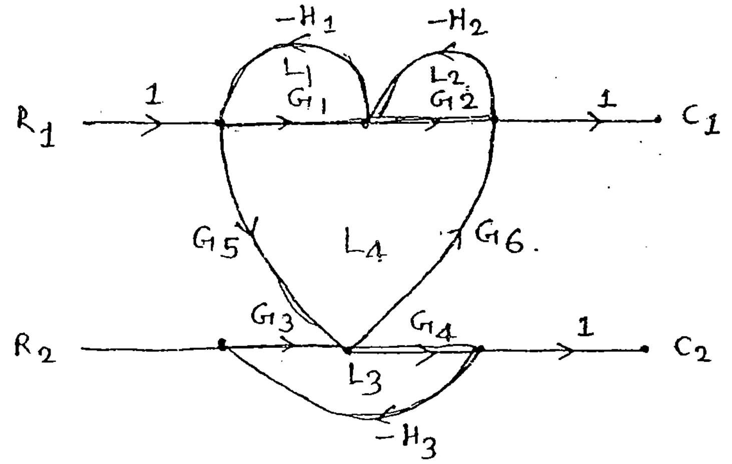
$$* \quad \frac{Y_5}{Y_4} = \frac{Y_5/Y_1}{Y_4/Y_1}$$

$$\frac{Y_4}{Y_1} = \frac{G_1G_2(1+H_4)}{\Delta}$$

$$*** \quad \boxed{\frac{Y_5}{Y_4} = G_3}$$

$$\frac{Y_5}{Y_3} = \frac{Y_5/Y_1}{Y_3/Y_1} = \frac{G_1G_2G_3\,(1+H_4)}{G_1(1+G_3H_2+H_4+G_3H_2H_4)}$$

* Find $\frac{C_1}{R_1}$, $\frac{C_1}{R_2}$, $\frac{C_2}{R_1}$, $\frac{C_2}{R_2}$ to the given multi-input multi o/p System.



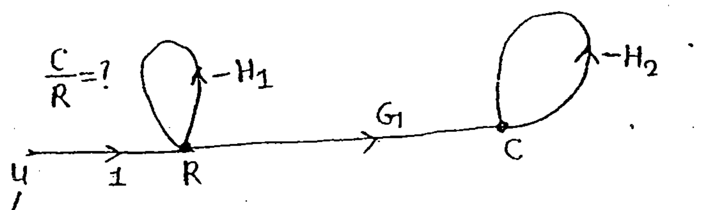
$$\left.\frac{C_1}{R_1}\right|_{\substack{C_2=0\\R_2=0}} = \frac{G_1G_2\,(1+G_3G_4H_3)+G_5G_6}{1+G_1H_1+G_2H_2+G_3G_4H_3-G_5G_6H_2H_1+G_1H_1G_3G_4H_3+G_2H_2G_3G_4H_3+G_2H_2G_3G_4H_3}$$

$$\left.\frac{C_1}{R_2}\right|_{\substack{R_1=0\\C_2=0}} = \frac{G_3G_6\,(1+G_1H_1)}{\Delta}$$

$$\left.\frac{C_2}{R_1}\right|_{R_2=C_1=0} = \frac{G_5G_4\,(1+G_2H_2)}{\Delta}$$

$$\left.\frac{C_2}{R_2}\right|_{R_1=C_1=0} = \frac{G_3G_4\,(1+G_1H_1+G_2H_2)}{\Delta}$$

Q. $\frac{C}{R}=?$



$$\frac{C}{R} = \frac{C/u}{R/u} = \frac{G_1}{1+H_2}.$$

u ↓ dummy node.

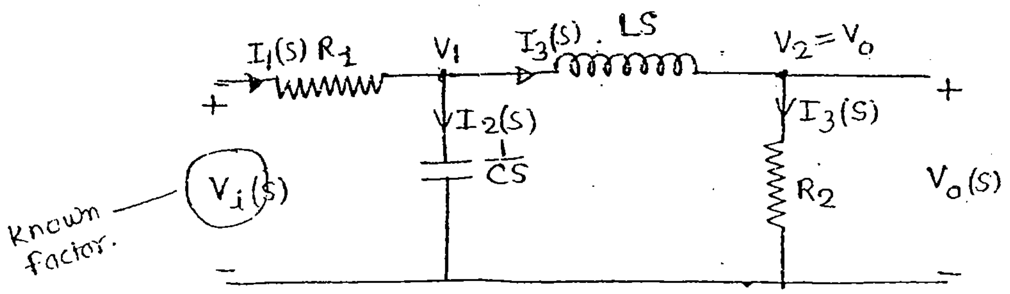
in the above SFG, the node are is not a i/p node because ... having incoming Branch, Hence we require to take

$$C = RG_1 - CH_2$$

$$C(1+H_2) = RG_1$$

$$\boxed{C/R = \frac{G_1}{1+H_2}}$$

* Construction of SFG to the Electrical Network:



* Select the Branch currents and node voltages.
* apply L.T. to the n/w variable and components.
* write the equations For unknown currents and voltages

$$I_1(S) = \frac{V_i(S) - V_1(S)}{R_1} \quad \text{——} (1)$$

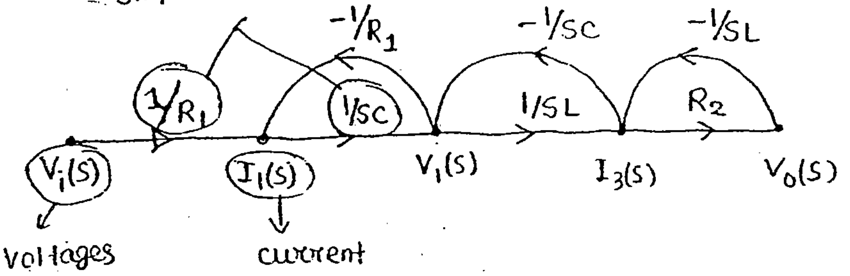
$$V_1(S) = I_2(S)\left(\frac{1}{CS}\right) = \frac{I_1(S) - I_3(S)}{CS} \quad \text{——} (2)$$

$$I_3(S) = \frac{V_1(S) - V_o(S)}{SL} \quad \text{——} (3)$$

$$V_o(S) = I_3(S) \cdot R_2 \quad \text{——} (4)$$

* * nodes in SFG nothing but Variable in Series Branch.
* (don't change sequence also)

Impedences or admittances.



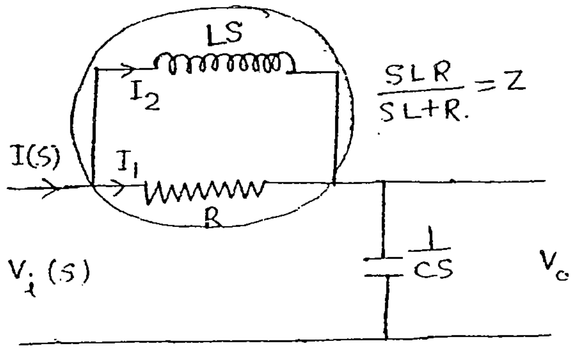
$$\boxed{(T/F)_{\substack{\text{Electrical} \\ n/w}} = T/F_{\text{(BD) or (SFG)}}}$$

→ Procedure to draw SFG directly

(*) Each element in a electrical n/w gives one Forward path and one negative F/b path from next successive nodes. Except the last element where we takes the o/p. the last element gives only forward path.

(*) take the Ratio of Impedence For series Branch element as a path gain and take the same Impedence for shunt Branch element as a path gain.

Q. construct the SFG to the given electrical n/w.

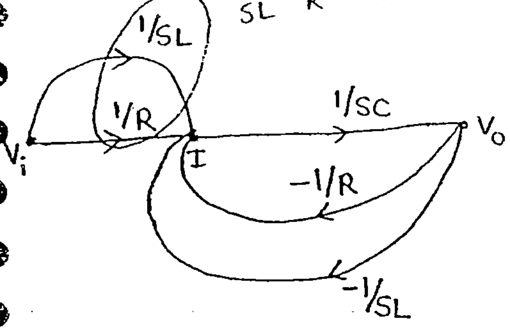


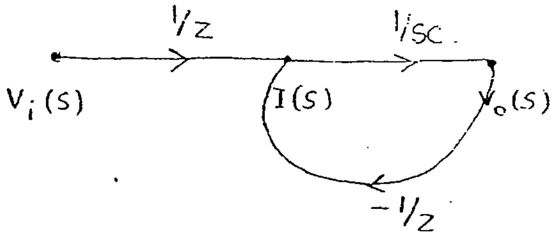
$$\frac{SLR}{SL+R} = Z$$

$$I = I_1 + I_2$$

$$I = \frac{V_i - V_o}{R} + \frac{V_i - V_o}{SL}$$

$$\frac{1}{SL} + \frac{1}{R} = \frac{SL+R}{SLR}$$





$$\frac{V_o(S)}{V_i(S)} = \frac{\left(\frac{SL+R}{SLR}\right)\frac{1}{SC}}{1 + \frac{(SL+R)}{S^2LCR}}$$

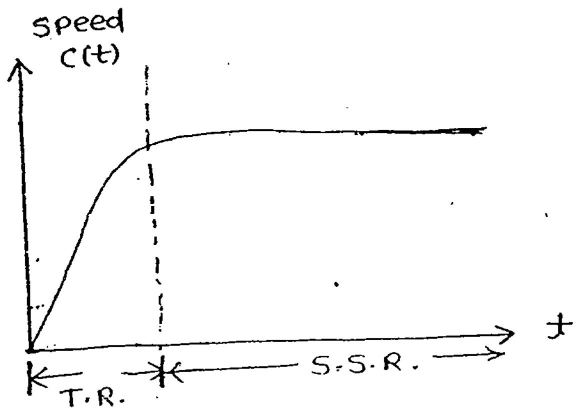
$$\frac{V_o(S)}{V_i(S)} = \frac{SL+R}{S^2LCR + SL + R}$$

→ Time domain Analysis :-

Purpose - to evaluate performance of System w.r.t. time

Time Response - If the Response of the System varies w.r.t. time then it is called time Response

→ Time Response = Transient Response + Steady-State Response.



$$c(t) = c_{tr}(t) + c_{ss}(t)$$

Q. $\underbrace{1 + 2\sin 2t + 3\cos 3t}_{} + \underbrace{4t\cancel{e^{-4t}} + 5e^{-5t}\sin 5t + 6t\cancel{e^{-6t}}\cos}_{}$ in the given tr.

Identify SS-transient and S.S. Response time Response.

any term consist exponential decay → (transient term.)

$$\boxed{\lim_{t\to\infty} c_{tr}(t) = 0}$$

Transient Response → it is a part of time Respone, that goes to zero as time becomes very large (∞).

means

$$\boxed{\lim_{t\to\infty} c_{tr}(t) = 0}$$

The terms which consist the exponential decay becomes the transient terms.

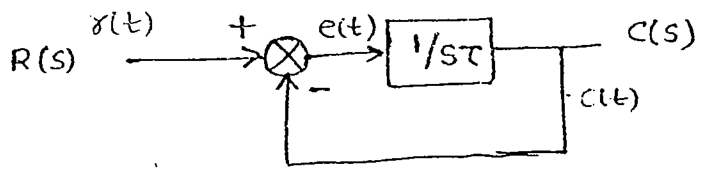
the poles which lies in left Half S-plane always give Transient term.

Steady State Response →

It is a part of the time Response that Remains after the transients becomes zero.
the poles which lies on Imagenary axis or Right Half S-Plane gives the steady State term.

* Time Response to the $\boxed{\text{first order System}}$ *

(RC or RL Ckt)



$$\frac{C(S)}{R(S)} = \frac{1}{\tau S + 1}$$

* Impulse Response

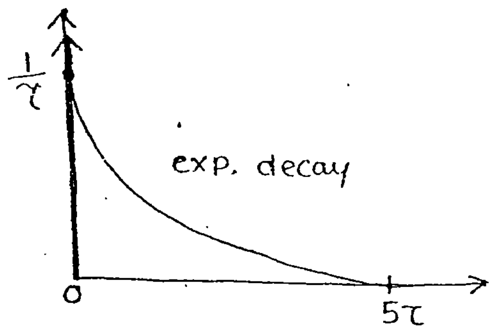
$$r(t) = \delta(t)$$

$$R(S) = 1$$

$$C(S) = \frac{1}{\tau(s + 1/\tau)} = \boxed{\frac{1}{\tau} e^{-t/\tau} = c(t)}$$

$$c(t) = \frac{1}{\tau} e^{-t/\tau}$$

* The Impulse Response Consist only transient term, the transient term Consist, the System Parameter, hence the Impulse Response is called System Response / Natural Response / Free forced Response.



Error –
deviation of o/p from the Reference i/p.

Steady State error means error at $t \to \infty$.
denoted as e_{ss}.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} [r(t) - c(t)] = \lim_{t \to \infty} \left(\delta(t) - \frac{1}{\tau} e^{-t/\tau}\right)$$

The Steady State error for Impulse i/p is not define because,

① Impulse Response not consist any S.S. term.

② there is no i/p at $t \to \infty$. Hence we can't compare the o/p with i/p.

* unit Step Response →

$$r(t) = u(t)$$

$$R(S) = 1/S.$$

$$\frac{C(S)}{R(S)} = \frac{1}{S\tau + 1}$$

$$C(S) = \frac{1}{\tau}\frac{1}{S}\left(\frac{1}{S + 1/\tau}\right)$$

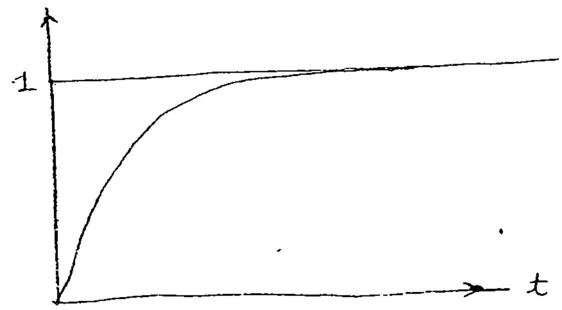
$$= \frac{1}{\tau}\left[\frac{1}{S} - \frac{1}{S + 1/\tau}\right]$$

$$c(t) = [\underbrace{1}_{\text{S-S}} - \underbrace{e^{-t/\tau}}_{\text{Tr. Res.}}]u(t)$$

i/p

System.

The S.S. Response depends on the i/p whereas, the transient Response depends on the system.



$$e_{ss} = \lim_{t \to \infty}\left[1 - (1 - e^{-t/\tau})\right]$$

$$e_{ss} = 0$$

* $\boxed{\text{o/p tracks the i/p.}}$

* unit Ramp Response →

$$r(t) = t\,u(t)$$

$$R(S) = 1/S^2$$

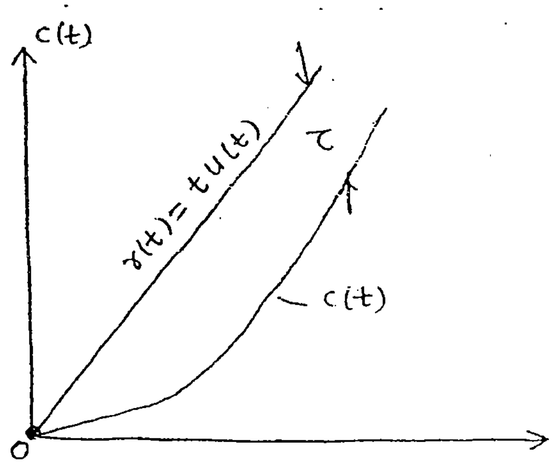
$$C(S) = \frac{1}{S^2(S\tau + 1)} = \frac{1}{S^2} - \frac{\tau}{S} + \frac{\tau^2}{\tau S + 1}$$

deviation

$$c(t) = \underbrace{(t - \tau)}_{ss} + \underbrace{\tau e^{-t/\tau}}_{Tr.}$$

$$e_{ss} = \lim_{t \to \infty} \left(\cancel{t} - \cancel{t} + \tau - \underbrace{\cancel{\tau e^{-t/\tau}}}_{0} \right)$$

$$= \tau.$$

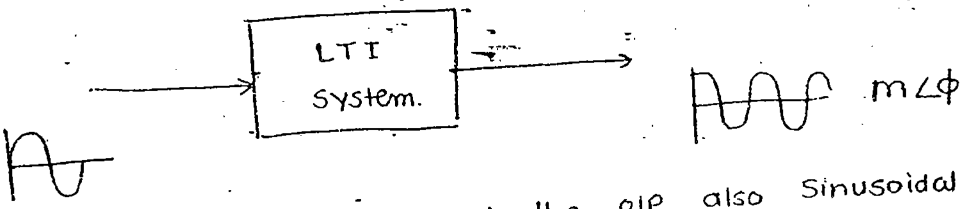


unit parabolic

$$r(t) = \frac{t^2}{2} u(t)$$

$$R(s) = 1/s^3.$$

Sinusoidal Response →



* For a LTI system if i/p is sinusoidal, the o/p also sinusoidal but difference in magnitude and phase. the std. form of i/p and o/p's are as follows.

$$r(t) = A\sin(\omega t \pm \theta) \Rightarrow c(t) = A \times m \sin(\omega t \pm \theta \pm \phi)$$

* A closed loop T.F. of a unity f/b system is

$$\frac{C(s)}{R(s)} = \frac{1}{s+1}$$

for i/p $r(t) = \sin t$ steady state Response is the sinusoidal o/p or

$$\omega = 1 \qquad s = j\omega = j1$$

$$\frac{C(s)}{R(s)} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}}\angle\phi = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$c(t) = 1 \times \frac{1}{\sqrt{2}}\sin(t - 45^\circ)$$

* Repeat the above problem.

$$\frac{C(s)}{R(s)} = \frac{s+2}{s+1} \qquad r(t) = 10\cos(2t + 45^\circ)$$

$$\frac{C(s)}{R(s)} = \frac{2+j2}{1+j2} = \frac{\sqrt{4+4}}{\sqrt{5}}\angle 45^\circ - \tan^{-1}2) = \frac{2\sqrt{2}}{\sqrt{5}}\angle 45 -$$

$$c(t) = 10 \times \frac{2\sqrt{2}}{\sqrt{5}}\cos(2t + 45^\circ + 45^\circ - \tan^{-1}2)$$

$$= 10 \times \sqrt{\frac{8}{5}}\cos(2t + 45^\circ - 18.43)$$

* A system with T.F.

$$\frac{Y(s)}{X(s)} = \frac{s}{s+P}$$

as an o/p.

$$Y(t) = \cos(2t - \pi/3)$$

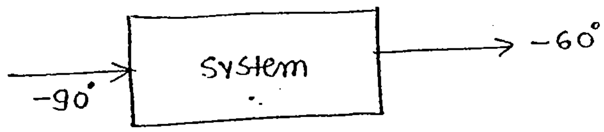
for the i/p signal

$$X(t) = P\cos(2t - \pi/2).$$

then system parameter $P = ?$

$\omega = 2.$

$$\frac{Y(S)}{X(S)} = \frac{Y(J\omega)}{X(J\omega)} = \frac{J2}{J2+P}.$$



$$\angle\left[\frac{Y(S)}{X(S)}\right] = +30^\circ$$

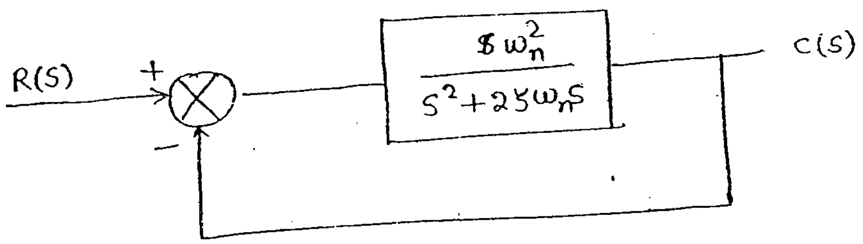
$$\frac{\angle S}{\angle(S+P)} = 30^\circ = \frac{\angle J\omega}{\angle(J\omega+P)} = 30^\circ$$

$$90 - \tan^{-1}\left(\frac{\omega}{P}\right) = 30^\circ$$

$$\tan^{-1}(2/P) = 60^\circ$$

$$\boxed{P = 2/\sqrt{3}}$$

* Time Response to the second order system.



$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}.$$

for System Stability

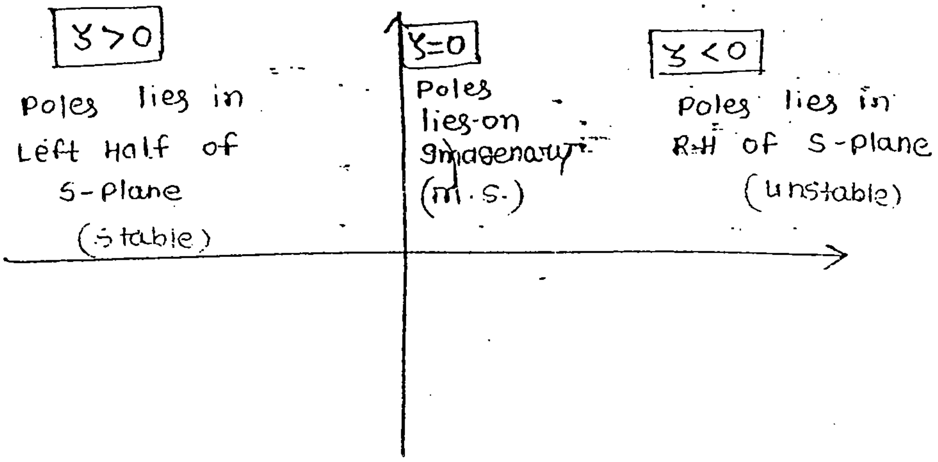
the 2nd order System nature is completely depends on ξ

* the 2nd order System is stable for all the +ve value

* the 2nd order sys. is because poles lies in the of $\xi < \infty$ and $\xi > 0$,

Left Half S-Plane.

$\zeta = 0$ poles are in imagenary (system m.s.)



Impulse Response

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

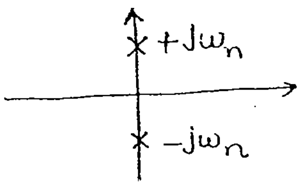
$$s = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$\longrightarrow \quad s \Rightarrow -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

case-1

$\zeta = 0$, (undamped System)

$$C(s) = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

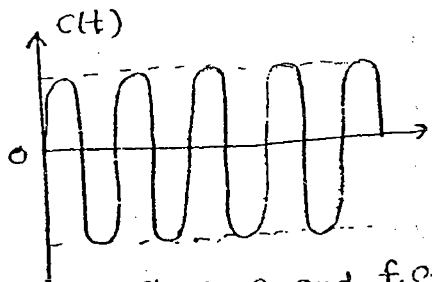


$T = \infty$.

freq. of oscillation $= \omega_n$ rad/sec.

marginal stable.

I.L.T. $\quad c(t) = \omega_n \sin\omega_n t$



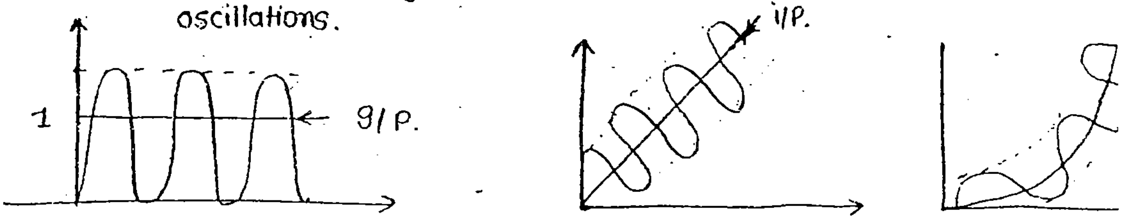
const. amp. and f.o.

undamped oscillation

* $\zeta=0$ * Poles on imaginary axis which are non-repeated.

the system is marginal stable.

the system Response is called constant amplitude and frequency of oscillations. which are called undamped oscillations.



* The 2nd order system nature is completely depends on ζ.

For example if

$\zeta=0$ the second order system nature is const. amplitude a Frequency of oscillation around the i/p which never b changed by changing the i/p, hence irrespective of any when $\zeta=0$ the second order system is called undamped

$0<\zeta<1$ underdamped system.

$\zeta=1$ critical damped system.

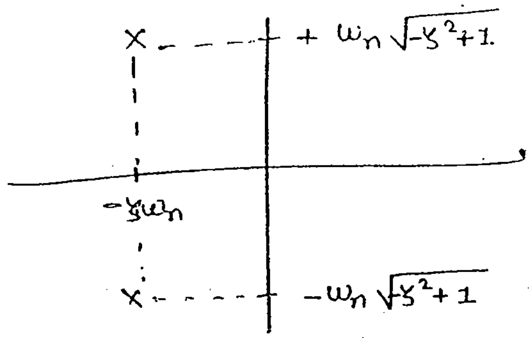
$\zeta>1$ overdamped system.

* when $\zeta=0$ Irrespective of all the i/p we can't find the steady state error because sys. is marginal stable. the steady state errors are valid for only closed loop stable system.

* when $\zeta>0$ and $\zeta<1$ * (underdamped system)

$$\longrightarrow \quad s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1} \quad (\zeta>1)$$

$$= -\zeta\omega_n \pm j\omega_n\sqrt{-\zeta^2+1} \quad (\zeta<1).$$



Stable system. $\begin{cases} \tau = 1/\zeta\omega_n \\ f.o.o. = \omega_n\sqrt{1-\zeta^2} \text{ rad/sec.} \end{cases}$

$$C(s) = \frac{\omega_n^2}{\left(s+\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}\right)\left(s+\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}\right)}$$

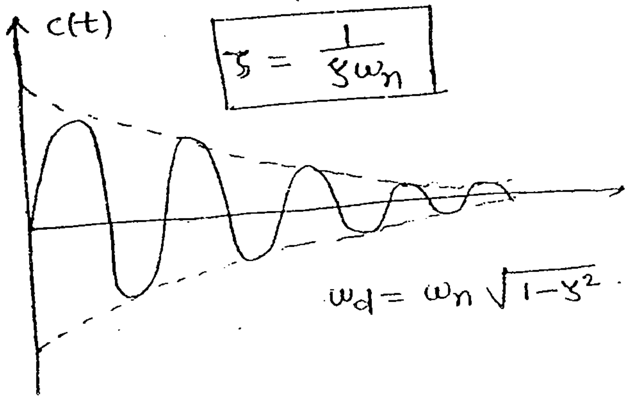
$$= \frac{\omega_n^2}{(s+\zeta\omega_n)^2 + \left(\omega_n\sqrt{1-\zeta^2}\right)^2}$$

$$= \frac{\frac{\omega_n}{\sqrt{1-\zeta^2}}\,\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2 + \left(\omega_n\sqrt{1-\zeta^2}\right)^2}$$

$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\right)t$$

T.C.

$$\boxed{\tau = \frac{1}{\zeta\omega_n}}$$



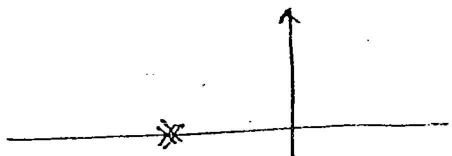
$\omega_d = \omega_n\sqrt{1-\zeta^2}$

* $0<\zeta<1$ then poles are complex conjugate in s-plane, system is stable, the system response is oscillation which exponential decay, frequency of oscillations are called damped oscillations.

any system gives the response of damped oscillation. It is called underdamped system.

* $\zeta = 1$ (critical damped)

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$



$s = -\omega_n$

Stable $\qquad \boxed{\tau = 1/\omega_n \text{ sec.}}$

relative stability decreases.

* order of time constant →

$\tau_{undamped} > \tau_{overdamped} > \tau_{underdamped} > \tau_{critical\ damped}$.

$\tau_{undamped}$	$\tau_{overdamped}$	$\tau_{underdamped}$	$\tau_{critical\ damped}$
(∞)	$\frac{1}{\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}}$	$\frac{1}{\zeta\omega_n}$	$1/\omega_n$
m.s.	largest τ so sluggish system or slow Response	medium τ	Lowest τ

Stable.

* unit step Response to the 2nd order System.

$r(t) = u(t)$

$R(s) = 1/s.$

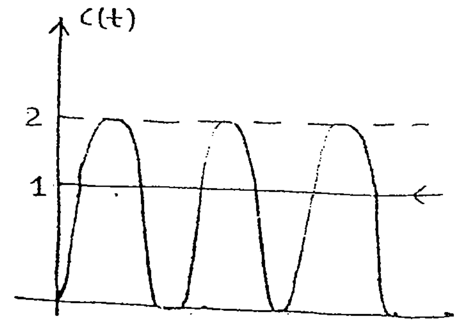
$$c(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

case-I → $[\zeta = 0$, undamped system$]$

$$c(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}$$

$$c(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

$$\boxed{c(t) = 1 - \cos\omega_n t}$$



Const. Amp. & F.O.O.

Undamped osc.

case-III ($\zeta=1$, critical damped sys.)

$$c(s) = \frac{\omega_n^2}{s(s^2+2\omega_n s+\omega_n^2)} = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$= \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

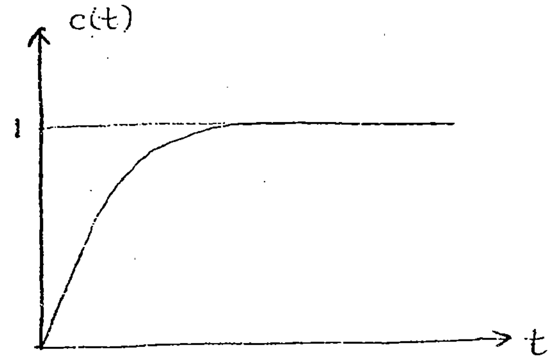
$$\omega_n^2 = A(s^2+2\zeta\omega_n s+\omega_n^2) + Bs(s+\omega_n) + Cs.$$

compare both side

[A = 1] [B = ...] [C = ...]

$$c(s) = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$c(t) = 1 - e^{-\omega_n t} - \omega_n t \cdot e^{-\omega_n t}$$



case-IV →

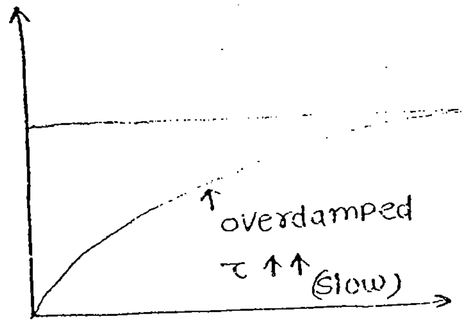
($\zeta > 1$) System is overdamped.

$$c(s) = \frac{\omega_n^2}{s\left(s+\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)\left(s+\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}\right)}$$

$$c(s) = \frac{1}{s} - \frac{K_1}{\left(s+\zeta\omega_n-\omega_n\sqrt{\zeta^2-1}\right)} - \frac{K_2}{\left(s+\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}\right)}$$

$$c(t) = 1 - \left[K_1 e^{-(\zeta\omega_n-\omega_n\sqrt{\zeta^2-1})} + K_2 e^{-(\zeta\omega_n+\omega_n\sqrt{\zeta^2-1})}\right]$$

$t = 0$ $\qquad$ $K_1 + K_2 = 1$



* For time domain analysis selected step i/p and underdamped system

		transient	Steady State	Stability
(Practically not possible)	Impulse	✓	X	✓
Practically exist	Step	✓	✓	✓
complete analysis of sys., widely used	Ramp	✓	✓	X } unst
	Parabolic	✓	✓	X

$$\% M_p = [c(t_p) - 1] \times 100\%$$

$$\% M_p = e^{-(n\pi\zeta/\sqrt{1-\zeta^2})} \times 100\% \quad (n=1 \text{ By default.})$$

$$\boxed{\% M_p = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100\%}$$

undershoot for 1^{st} valley point.

$$\% M_p = e^{-(2\zeta\pi/\sqrt{1-\zeta^2})} \times 100\%$$

Settling time

It is the time required for response to rise and reach the specified tolerance band usually $\pm 2\%$ or 5%.

$$\pm 5\% \qquad t_s = 3\tau = \frac{3}{\zeta\omega_n}$$

$$\pm 2\% \qquad t_s = 4\tau = \frac{4}{\zeta\omega_n}$$

$$0\% \text{ (s.s)} \qquad t_s = 5\tau = \frac{5}{\zeta\omega_n}$$

* Time period of the oscillation

$$T_{osc.} = \frac{2\pi}{\omega_d} = \underline{\underline{2t_p}}.$$

* No. of oscillations before Reaching s.s.

$$N = \frac{t_s\,(\pm 2\% \text{ or } \pm 5\%)}{T_{osc.}} = \frac{t_s}{2\pi/\omega_d} = \frac{t_s}{2t_p}.$$

* the unit step Response of the system is shown in the fig. Find the following factors.

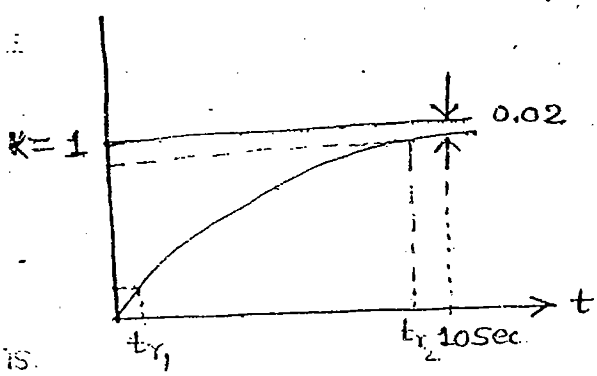
① Time constant.

② delay time

③ Rise time

④ Peak time and peak overshoot.

① $t_s = 4\tau$
$10 = 4\tau$ $\Big\}$ $\pm 2\%$
$\boxed{\tau = 2.5 \text{ Sec.}}$



* Std. form of unit step Response is.

$$c(t) = \underbrace{K}_{\text{S.S. value}}(1 - e^{-t/\tau})$$

$$c(t) = 1 - e^{-t/\tau}$$

at $t = t_d \implies c(t) = 0.5$

$$0.5 = 1 - e^{-t_d/2.5}$$

$$0.5 = e^{-t_d/2.5}$$

$$\ln(0.5) = -t_d/2.5$$

$$\boxed{t_d = 1.732 \text{ Sec.}}$$

* For exp. Rise Signal the duration for rise time is 10% to 90%

$$t_r = t_{r_2} - t_{r_1}$$

$t_{r_2} \Downarrow c(t) = 0.9$, $t_{r_1} \Downarrow c(t) = 0.1$

$$\boxed{t_r = 2.2\tau}$$

$$\boxed{t_r = 5.5 \text{ Sec.}}$$

The system Response not consist the any peak hence no peak time and no overshoot.

* The Impulse Response of a unity f/b control system is

$$c(t) = e^{\overset{R.P.}{-3}t} \sin \overset{Im.P.}{4}t$$

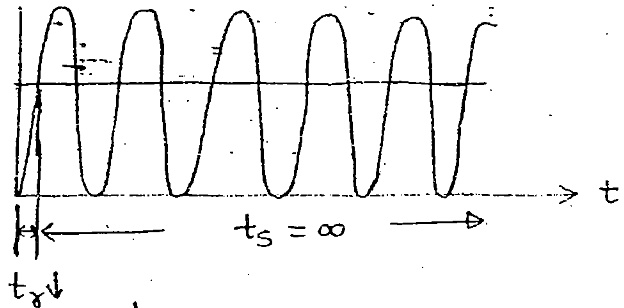
Find the following factor.
① Peak time ② settling time ③ damping factor ζ
④ ω_n ⑤ t_d ⑥ t_r ⑦ M_p.

$\zeta = 0$ (undamped)

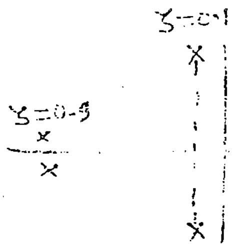
For Smooth Response

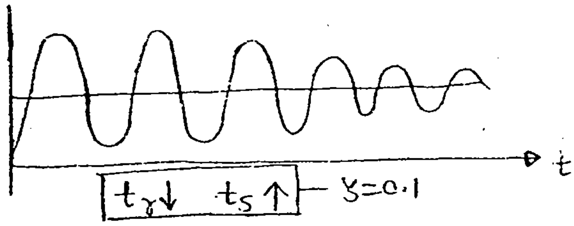
→ $t_r \downarrow$ $t_s \downarrow$

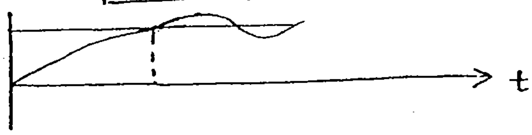
→ $0.4 < \zeta < 0.7$



rise time — Smo
Settling time larg







$\boxed{\begin{matrix} t_r \downarrow \\ t_s \downarrow \end{matrix}}$ $\zeta =$

for time domain specification select underdamped sys. because if select critical or overdamped sys. the rise time is very-very large but settling time is small, whereas if select the undamped system then rise time very-very small but settling time $= \infty$.

Practically any system required smallest t_r and smallest t_s which is not possible in critical, over and undamped system.

* If ζ selected b/w 0.4 to 0.7 then we can get the medium values of t_r and t_s.

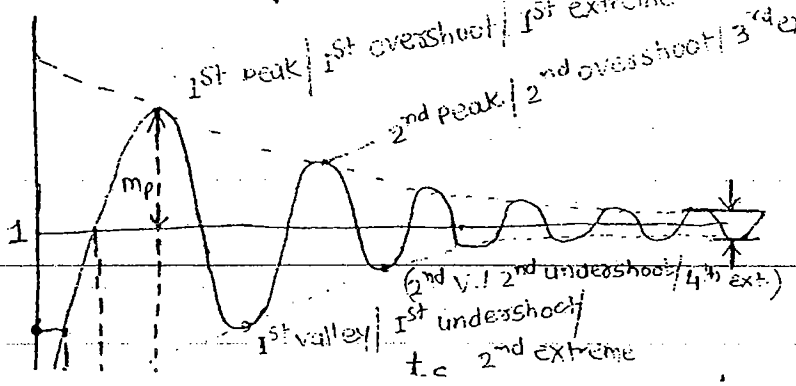
Practically to implement any system ζ selected b/w 0.4 to 0.7.

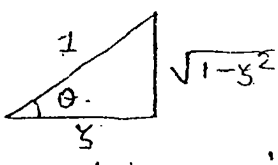
Time domain Specification—

when ($\zeta > 0$ and $\zeta < 1$) the unit step response of system is

$$c(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\left(\omega_n \sqrt{1-\zeta^2}\right)t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

$\cos^{-1}\zeta$





allowable tolerance $\pm 2\%$ to $\pm 5\%$

0 to 100% underdamped

5 to 95% critical

10 to 90% overdamped.

delay time – It is the time required for response to rise from 0 to 50% of the final value.

$$\boxed{t_d = \frac{1+0.7\zeta}{\omega_n}} \text{ sec.}$$

Rise time - It is the time required for response to rise from 0 to 100% for underdamped, 5 to 95% for critical damp, 10 to 90% for overdamped.

$$\boxed{t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_d}} \text{ sec.}$$

$$\boxed{t_r = \frac{\pi - \cos^{-1}(\zeta)}{\omega_d}} \text{ sec.}$$

Radian – (calculator)

Peak time It is the time required for response to rise and reach the peaks of time Response.

$$t_p = \frac{n\pi}{\omega_d} \quad \left[\begin{matrix} n = 1 & \text{By default} \\ 1^{st} \text{ peak} & \end{matrix}\right]$$

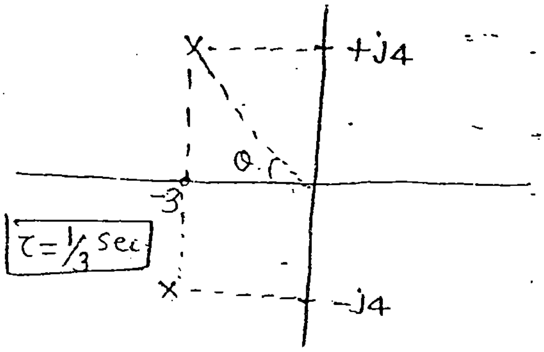
$$\boxed{t_p = \frac{\pi}{\omega_d}} \text{ sec.}$$

2^{nd} Peak $\quad t_p = \frac{3\pi}{\omega_d}$ sec.

1^{st} valley $\quad t_p = \frac{2\pi}{\omega_d}$ sec.

* Peak overshoot → It gives the normalized difference b/w time response peak to steady state value..

$$\% \, M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$



$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{4}$ s

$\pm 2\%$ $t_s = 4\tau = 4/3$

$\zeta = \cos\theta$.

$\boxed{\zeta = 3/5 = 0.6}$

$\zeta\omega_n = 3$

$\boxed{\omega_n = \frac{3}{0.6} = 5 \text{ rad/}}$

$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_d} = 0.55$ sec.

$t_d = \frac{1+0.7\zeta}{\omega_n}$

$\% \, m_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% = 9.4\%$

$\boxed{t_d = 0.284 \text{ sec.}}$

IInd method –

$$C.L.T.F. = L[\text{Impulse Response}]$$

$$= \frac{4}{(s+3)^2 + 4^2}$$

$$= \frac{4}{s^2 + 6s + 25}$$

$$= \frac{4}{25}\left(\frac{25}{s^2+6s+25}\right)$$

$\frac{4}{25}$ → affect only S.S. value
→ But not any time domain Specification.

* Find the % of peak overshoot to the following System.

$$\frac{C(s)}{R(s)} = \frac{25}{s^2 + 25}$$

⟶ $\zeta = 0$, undamped → $\% \, m_p = e^{-\pi \times 0/\sqrt{1-0}} = 100\%$

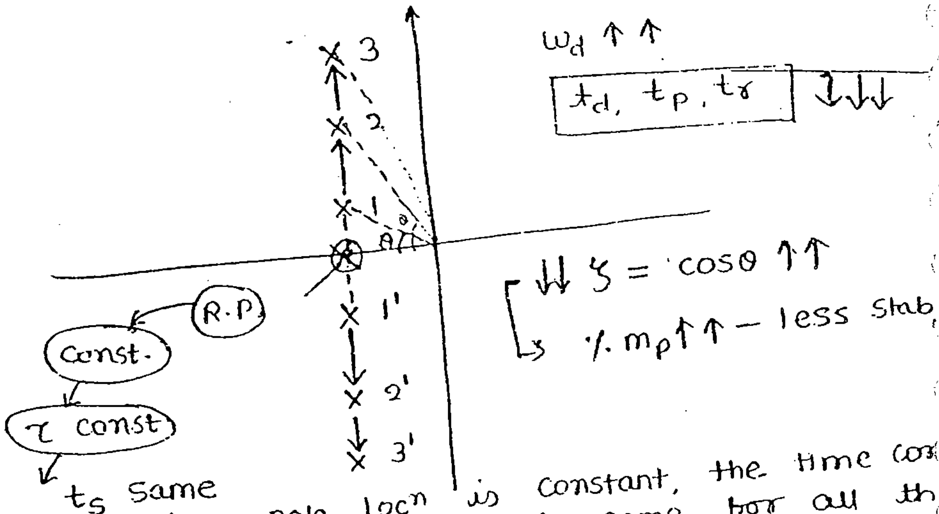
$$C(s) = \frac{100}{s^2 + 20s + 100}$$

⟶ $\zeta = 1$, Critical damped, $\% \, m_p = e^{-\pi \times 1/\sqrt{1-1^2}} \times 100\% = 0$.

Note – when ζ increase from 0 to 1, the % of peak overshoot decrease from 100% to 0%.

→ when $\xi \geq 1$ and increased then ... is 0%. Because there is no oscillation in the system response.

*→ Find the variations in time domain specifications to the given poles path in the s-plane.

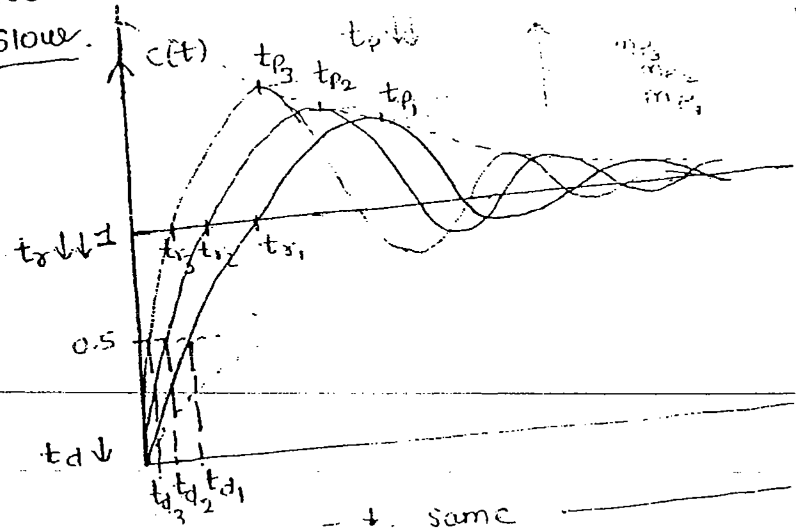


* as the Real part of the pole locn is constant, the time constant is constant, Hence the settling time is same for all the poles location.

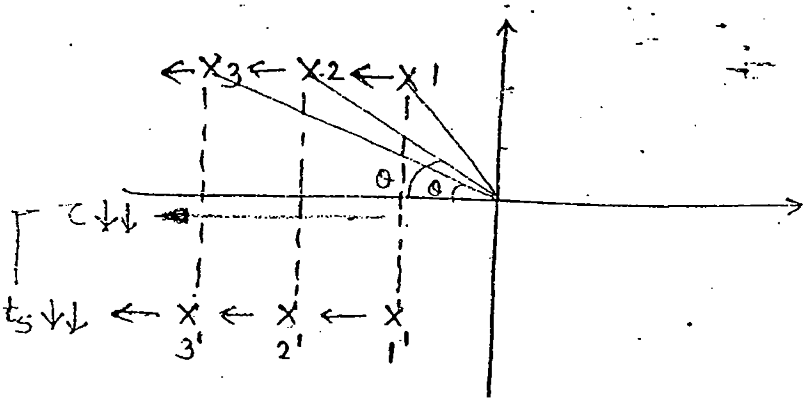
* as the imagenary part is increases the damped osc. ω_d must be increases. as $\omega_d \uparrow$ the time specification (delay, rise and peak time) ↓.

* as the inclination of the pole θ increases, the damping factor ξ decreases. Hence the % of peak overshoot decreases so that system becomes less stable

the optimum value of $\boxed{\% m_p = 5\% \text{ to } 40\%}$ if the % of overshoot is more than 40% system become less stable if the peak overshoot is less than 5% system response becomes very slow.



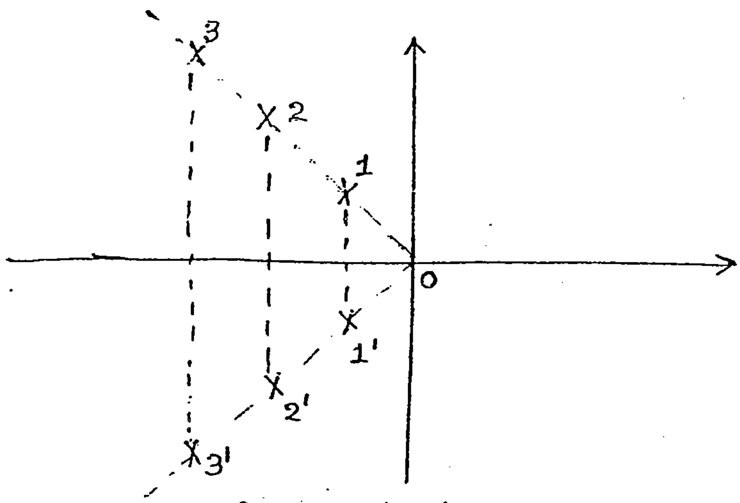
Horizontal movement of the poles.



as the poles are moving towards left the time Constant decreases, Hence settling time also decreases.

* as the Imagenary part = Const.
  damped oscillations. ω_d = constant.
  Hence Peak time t_p = Constant
  But there exist Slight variation in t_d and t_r
* as the inclination of the pole decreases, then dampin factor ξ ↑↑ Hence,
  % m_p is decreases. so that System becomes more Stable.

Diagonal movement of the poles →



as the inclination of pole θ = constant
damping factor ξ = constant
Hence % m_p = constant
So there is no effect on Steady State error.
∴ no effect on sys stability

as img. part ↑ then damped oscillation ω_d ↑

Hence t_d, t_r, t_p = ↓

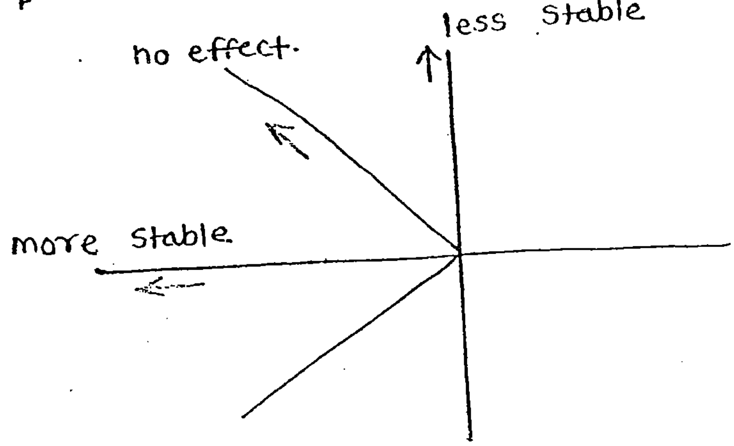
poles are moving left then time const. ↓ Settling time ↓

* Final conclusion.

$t_s \rightarrow$ R.P. of pole (τ)

$t_p \rightarrow$ Imagenary part of pole (ω_d)

$\% M_p \rightarrow \zeta$.



* Find the time domain specification to the given unity F/b system.

$$G(s) = \frac{25}{s(s+4)} \qquad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25}$$

$\boxed{\omega_n = 5 \text{ rad/sec.}}$

$\boxed{\zeta = 0.4}$ $\qquad$ $\boxed{\omega_d = 4.5 \text{ rad/sec.}}$

$$t_p = \frac{\pi}{\omega_d} = 0.698 \text{ sec.}$$

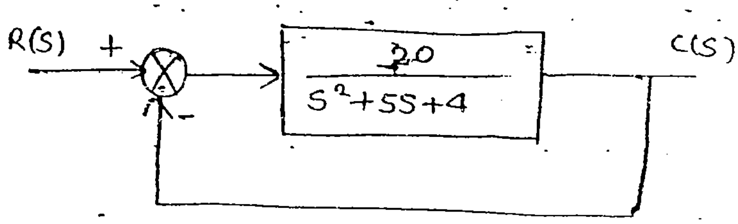
$$t_d = \frac{1+0.7\zeta}{\omega_n} = 0.256 \text{ sec.}$$

$$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_d} = 0.44 \text{ sec.}$$

$$\% M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% = 25\%$$

$$\frac{4}{\ldots} = 2 \text{ Sec.}$$

Q. (*) Find the time domain specification to the given unity f/b system.



$$\frac{C(S)}{R(S)} = \frac{20}{S^2+5S+24}$$

$$= \left(\frac{20}{24}\right)\frac{24}{S^2+5S+24}.$$

→ effect the S.S. value But not time domain specification

$\omega_n = 4.89$ rad/sec.

$\zeta = 0.511$

$\omega_d = 4.2$ rad/sec.

*

$t_p = \frac{\pi}{\omega_d} = 0.747$ sec.

$t_d = \frac{1+0.7\zeta}{\omega_n} = 0.277$ sec.

$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_d} = 0.50$ sec.

$\% M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 15.44\%$.

$t_s = \frac{4}{\zeta\omega_n} = 1.6$ sec.

(b) find the unit step Response to the above System.

$$c(t) = 1 - \frac{e^{-2.5t}}{\sqrt{1-0.511^2}} \sin\left(4.2t + \cos^{-1}0.511\right)$$

$$= 1 - \frac{e^{-2.5t}}{}$$

Variations in time domain specification w.r.t. ζ

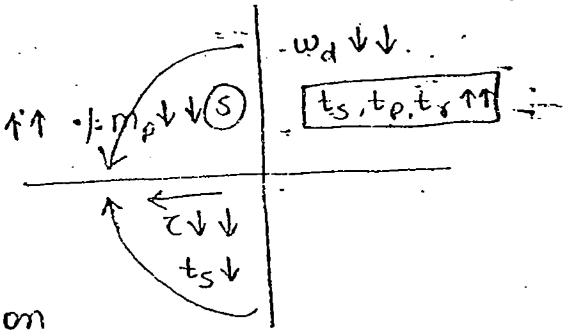
as $\zeta\uparrow$ from 0 to 1 → the poles moves towards the left and near to the Real axis.

in this case time const ↓

settling time ↓

as $\omega_d \downarrow$ then $t_d, t_p, t_r \uparrow\uparrow$

→ as $\zeta \uparrow$
$\% m_p \downarrow$
then System become
more stable



Find the time domain specification where to the following system where x is i/p, y is o/p.

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 8x.$$

$$\frac{Y(S)}{X(S)} = \frac{8}{s^2+4s+8}$$

$$\boxed{\omega_n = 2\sqrt{2} \text{ rad/sec.}}$$

$2\zeta\omega_n = 4$ 2
$\zeta = 1/\sqrt{2} = 0$

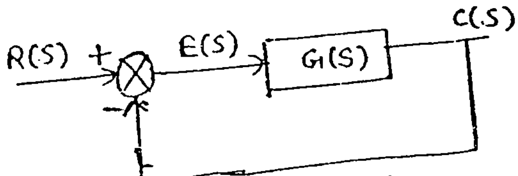
✳ Steady State error →

error → deviation of the o/p from the i/p.

S.S. error → the error at $t \to \infty$. , i.e

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(S)$$

Let consider the unity f/b system



✳✳✳

$$\boxed{\lim_{s\to 0} \frac{s\,R(S)}{1+G(S)} = e_{ss}.}$$

$$\frac{E(s)}{R(S)} = \frac{R(S)}{R(S)} - \frac{C(S)}{R(S)}$$

$$1 - \frac{C(S)}{R(S)} = \frac{1}{1+G}$$

Steady state error depend on two bactor $\rightarrow$

type of the i/p

type of the system.

$\rightarrow$ the Steady State errors are calculated to only closed loo[p] Stable Systems.

$\rightarrow$ The Steady State errors are valid bor only unity f/b Sys.

$\rightarrow$ If non unity f/b Sys. given, It should by Converted into unity f/b.

* $\boxed{\text{Type of the i/p}}$ $\rightarrow$

	0. Step	1. Ramp	2. Parabolic
$r(t)$ (Input)	$At^0u(t)$	$At^1u(t)$	$A\frac{t^2}{2}u(t)$
e_{ss}	$\frac{A}{1+k_p}$	$\frac{A}{K_v}$	$\frac{A}{K_a}$
error Constant	$K_p \rightarrow$ Position error constant $\lim_{S\to 0} G(S)$	$K_v \rightarrow$ Velocity error constant $\lim_{S\to 0} SG(S)$	$K_a \rightarrow$ acceler[ation] error const $\lim_{S\to 0} S^2G(S)$

* TYPE OF THE SYSTEM $\rightarrow$

the Standard form of system is represented as.

$$G(S)\,H(S) = \frac{K(1+ST_1)(1+ST_2)\cdots}{S^{n}(1+ST_a)(1+S\tau_b)\cdots}$$

S^n — type – n sys.

Consider step i/p and dibberent types of the systems.

0. step i/p, $e_{ss} = \frac{A}{1+k_p}$

* TYPE - 0 SYSTEM

$$K_p = \lim_{S\to 0} \frac{K(1+ST_1)(1+ST_2)\cdots}{S^0(1+S\tau_a)(1+S\tau_b)} = K$$

$$e_{ss} = \frac{A}{1+K} = \text{constant}$$

TYPE-1 SYSTEM

$$K_p = \lim_{s\to 0} \frac{K(1+s\tau_1)(1+s\tau_2)}{s^1(1+s\tau_a)(1+s\tau_2)} = \infty.$$

$$e_{ss} = \frac{A}{1+K_p} = 0.$$

Type =	i/p	$\Rightarrow$	e_{ss} constant
Type >	i/p	$\Rightarrow$	0 (Big zero).
Type <	i/p	$\Rightarrow$	∞

The Steady State errors are required to calculate only in 3 cases.

① Type 0 and Step i/p.

② type 1 and Ramp i/p

③ type ② and parabolic i/p

Remain all the cases the e_{ss} becomes either 0 or ∞.

→ $$G(s) = \frac{10(s+1)}{s^2(s+2)(s+10)}$$

$$r(t) = \left(10t^0 + 5t^1 + \frac{t^2}{2}\right)u(t)$$

Type		i/p		e_{ss}
2	>	0		0 +
2	>	1		0 +
2	=	2	—	$A/K = 1/0.2 = 2$

$e_{ss} = 2$

$$k = \frac{10\times 1}{2\times 10} = \left(\frac{\text{Nu. const.}}{\text{Deno. const.}}\right) = 0.5$$

* * * Find the e_{ss} to the given unity f/b System for the following i/p

$$G(s) = \frac{10}{s(s+2)}$$

① $10\,u(t)$

② $10t\,u(t)$

③ $10t^2\,u(t)$

④ $(1+t)\,u(t)$

⑤ $(1+t+t^2)\,u(t)$

① $R(s) = 10/s$

① $10\,t^0\,u(t)$ — 0

② $10\,t^1\,u(t)$ — $A/K = \frac{10}{10/2} = 2$

③ $10\,t^2\,u(t)$ — ∞

④ $(1+t^1)\,u(t)$ — $0 + \frac{1}{10/2} = 0.2$

⑤ $(1+t+t^2)\,u(t) = \infty.$

→ $G(s) = \frac{(s+1)}{s^2(s+2)(s+10)}$, $H(s) = 1$

① 0

② 0

$\frac{\overset{A}{20}t^2}{2}$ ③ $\frac{20}{\left(\frac{1}{2\times 10}\right)} = 400$

④ $0+0 = 0$

$\overset{A}{2}\left(t^2/2\right)$ ⑤ $0+0+\frac{2}{(1/20)} = 40.$

Q. Repeat above problem.

$$G(s) = \frac{1}{s^2\,(s+2)\,(s+10)} \qquad H(s) = 1.$$

$$\frac{C(s)}{R(s)} = \frac{1}{s^4+12s^3+20s^2+\_\_+1}$$

(s^1 term missing)

CLTF → add the numerator term into denominator

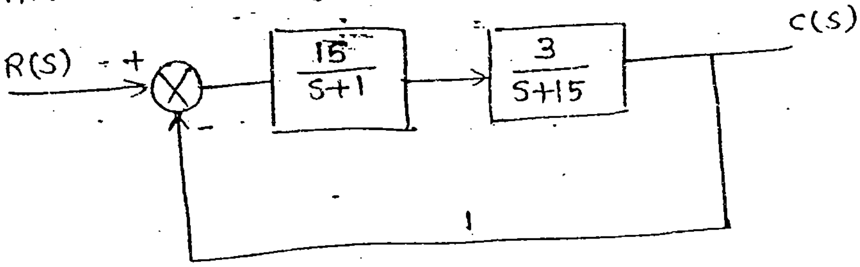
So System is not stable (either it may be m.s. or unstable).

Note- the above System is unstable for CL, Hence we can't find e_{ss}.

the $\boxed{e_{ss} \text{ calculated to only closed loop system}}$

the ~~close loop system Stability is verified by using~~ R-H criteria.

* Find the e_{ss} to the following system. to the unit step i/P.



$$\frac{C(S)}{R(S)} = \frac{45}{(S+1)\ (S+15)+45}$$

$$C(S) = \frac{45}{S[(S+1)\ (S+15)+45]}$$

$$e_{ss} = \lim_{S \to 0} \frac{\cancel{S} \times 1/\cancel{S}}{1 + \frac{45}{(S+1)\ (S+15)}} = 1/4$$

* to calculate the e_{ss} required the O.L.T.F. But calculated to Closed loop System.

$$e_{ss} = \frac{A}{1+K} = \frac{1}{1+\frac{45}{15}} = 1/4$$

* The O.L.T.F. of a ~~open loop~~ unity f/b sys. is

$$G(S) = \frac{K}{S(S+1)\ (S+2)}$$

the $e_{ss} = 0.1$ the value of K is for unit Ramp i/P.

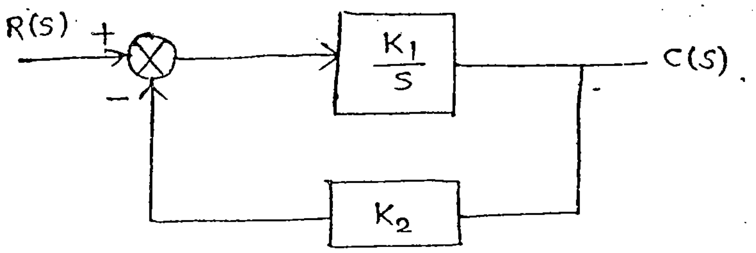
$$0.1 = \lim$$

*

* $0.1 = A/K = 1/K/2 =$

$$\boxed{K = 20}$$

Q. For the system shown in figure steady state gain = 4, Time constant = 0.2 sec. the values of K_1 and K_2 are.



$$\frac{C(S)}{R(S)} = \frac{K_1/S}{1+\frac{K_1K_2}{S}} = \frac{K_1}{S+K_1K_2}$$

$$= \frac{K_1}{K_1K_2\left(S/K_1K_2+1\right)} = \frac{1/K_2}{\left(S/K_1K_2+1\right)}$$

$$\frac{C(S)}{R(S)} = \frac{K}{S\tau+1}$$ (If sys. gain or s.s. gain given otherwise K=1)

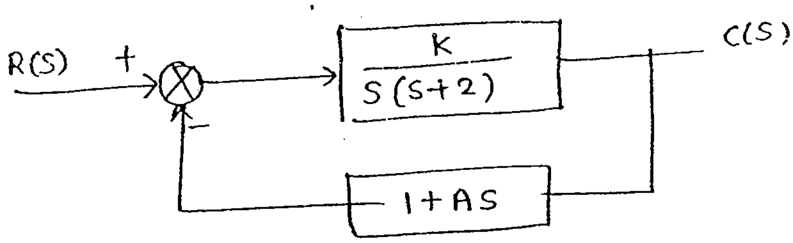
$$= \frac{4}{S(0.2)+1}$$

$1/K_2 = 4$

$\boxed{K_2 = 0.25}$

$\boxed{K_1 = 20}$

* For the system shown in figure the undamped freq. of osc. 4 rad/sec., damping factor is 0.7 the values of K and A are



$$\frac{C(S)}{R(S)} = \frac{K}{S(S+2)+\frac{K}{\cancel{S(S+2)}}(1+AS)}$$

$$\Rightarrow \frac{K}{S^2+2S+KAS+K} = \frac{K}{S^2+S(2+KA)+K}$$

$$= \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}.$$

comparision

$$2 + KA = 5.6$$

$$KA = 3.6$$

$$A = \frac{3.6}{16}$$

$$\boxed{A = 0.225}$$

* the O.L.T.F. of a unity f/b system is G(S) the $e_{ss} = 0$ for

(a)	Step i/p's	type 0 System.	A/1+K
(b)	Step i/p's	type 1 System.	0
(c)	Ramp i/p	type 0 System.	∞
(d)	Ramp i/p	type 0 System.	A/K.

* The control System described by following D.E.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 10(1 - e^{-2t})$$ the Response at $t \to \infty$.

* $\lim_{t \to \infty} y(t)$

$$\frac{Y(S)}{X(S)} = \frac{10 \times 2}{s(s+2)\ (s^2+2s+5)}$$

$$\lim_{s \to 0} \cdot \frac{\cancel{s} \times 10 \times 2}{\cancel{s}(s+2)\ (s^2+2s+5)} = 2.$$

* The Laplace transform $f(t) \longleftrightarrow F(S)$

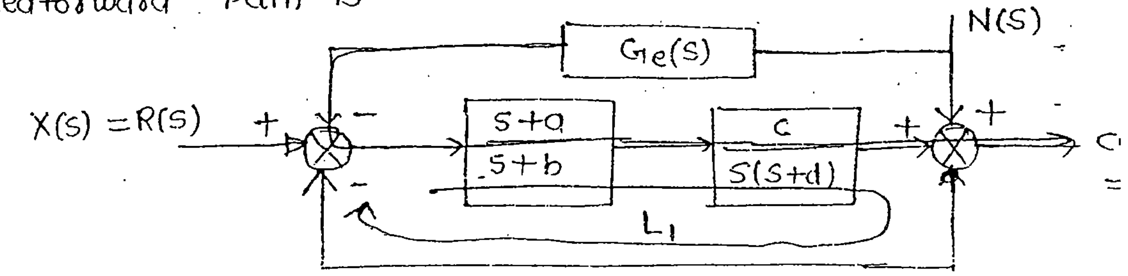
$$F(S) = \frac{\omega}{s^2 + \omega^2}$$

Never apply Final value theorem for Sine and cosine term because It maintain oscillation at $t \to \infty$.

Ans. → $\boxed{-1 \text{ to } +1}$

(*) For a LTI system shown in figure X(S) is I/P
Y(S) is O/P.

In order to nullify the effect of noise the gain of feedforward path is



$\underline{\text{Sol}^n}$

$$\left.\frac{Y(S)}{N(S)}\right|_{X(S)=0} = \frac{1 - G_e(S)\left[\frac{(S+a)}{(S+b)}\cdot\frac{c}{S(S+d)}\right]}{1 + \frac{c(S+a)}{S(S+b)(S+d)}}$$

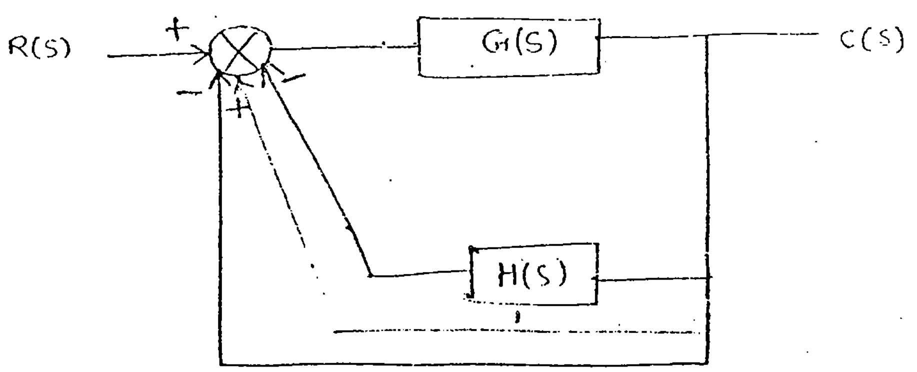
$$= 0$$

$$\boxed{G_e(S) = \frac{S(S+b)(S+d)}{c(S+a)}}$$

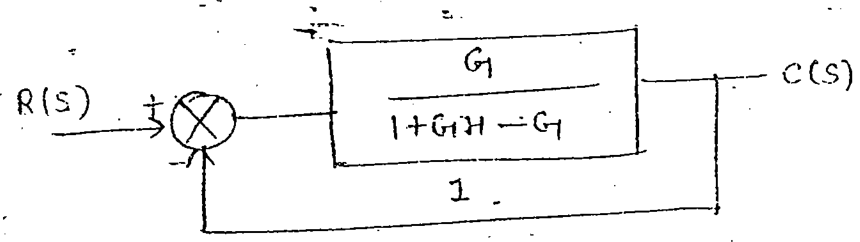
* Steady state errors to the non unity f/b system –

e_{ss} calculated for $H(S) = 1$

→ If non unity f/b system given, it should be converted into unity f/b as follows.

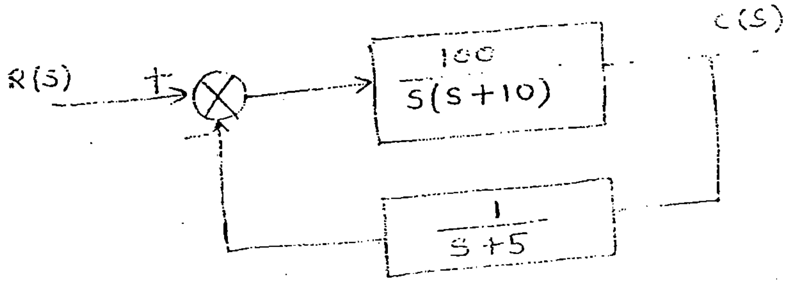


$G_{NUF}(s)$ = equivalent O.L.T.F. for non unity f/b.



$$\boxed{G_{NUF}(s) = \frac{G}{1+GH-G}}$$

* For the system shown in figure the $e_{ss} = ?$ for unit step I/P.



① CLTF = $\frac{G}{1+GH}$

② Find $G_{NUF}(s) = \frac{G}{1+GH-G}$ by subtracting numerator in the denominator

③ compare g/p and type, get solution

$$\frac{C(s)}{R(s)} = \frac{100}{s(s+10)+\left(100/(s+5)\right)} = \frac{100\,(s+5)}{s^3+15s^2+50s+1}$$

$$G_{NUF(s)} = \frac{100\,(s+5)}{s^3+15s^2+50s+100-100s-500}$$

$$= \frac{100\,(s+5)}{s^3+15s^2-50s-400}$$

$$e_{ss} = \frac{A}{1+K} = \frac{1}{1+\left(\frac{500}{-400}\right)} = -4$$

* Stability *

The LTI system is said to be stable if satisfies the following conditions.

① If the system excited by a Bounded i/p the o/p must be Bounded.

② If i/p to the system is zero, the o/p must be zero irrespective of all the initial conditions.

The stability are classified into two ways.

→ Based on operating condition.

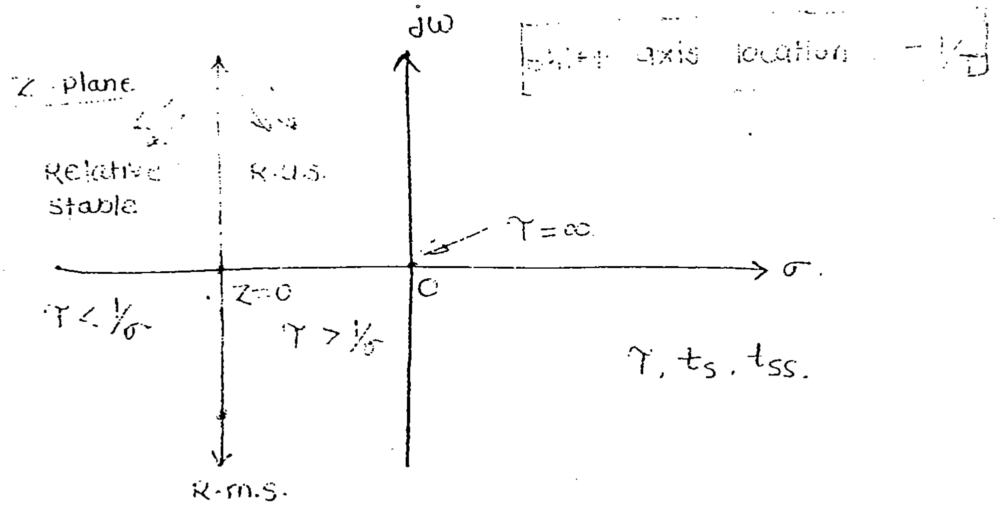
*① Absolute Stable System. → Here system is stable for all the value of sys. parameter like K from 0 to ∞.

*② conditional stable system – Here system is stable for certain Range of sys. parameters like K from 0 to 100.

*③ marginal / critical / limitedly stable system – A sys. is said to be m.s. if it gets for Bounded i/p the o/p maintains constant amplitude and freq. of osc.

④ Relative Stability :-

Relative stability is applicable for closed loop stable systems only.



τ, t_s, t_{ss}.

By using Relative Stability concept we can find time const. Settling time, time Required to Reach steady state.

Routh- Hurwitz Criteria

Purpose - * To find closed loop Sys. Stability

* to find no. of Poles in the left, Right, or an Imaginary axis of the S-Plane
* to find the Range of K-value. for System Stability.
* to find the K-value, to become the system m.s.
* to find the frequency of oscillation when system is marginal stable
* ~~too~~ find the Relative Stability.

By using R.S. concept we can find sys., time Const. , settling time, and time Required to Reach Steady State.

To find the closed loop system stability by using R-H criteria required a characteristic equation. whereas in remain all the Stability techniques, required O.L.T.F.

The n^{th} order general form of char. Equation is.

$$= a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \cdots + a_{n-1} s^1 + ($$

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \; \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \; \cdots \\
s^{n-2} & \dfrac{a_1 a_2 - a_0 a_3}{a_1} & \dfrac{a_1 a_4 - a_0 a_5}{a_1} & & \\
\vdots & & & & \\
s^0 & & & &
\end{array}$$

* The conditions for system stabilities are

(a) all the co-efficients in the first column should Have same sign and no coefficient should zero.

(b) If any sign change occur in first column then System is unstable. The no. of sign changes in the first Colomn = no. of poles in the Right Half of S-plane.

* Find the stability to the following characteristic equation

(1) $s+10=0$ (S)

(2) $s^2+25=0$ (ms)

(3) $s^2+10s+100=0$. (S)

(4) $s^3+8s^2+9s+10=0$. $(72>10)$ (S)

(5) $s^3+6s^2+4s+100=0$. $(24<100)$ (us)

(6) $s^3+8s^2+4s+32=0$. $(32=32)$ (m.s.)

* CE $\Rightarrow$ $as^1+b=0$

s^1	1(a)
s^0	10(b)

$a, b > 0$ or $a, b < 0$. (Stable)

* CE $\Rightarrow$ $as^2+bs+c=0$

s^2	a	c
s^1	b	0
s^0	c	

$a, b, c > 0$ or < 0, (Stable)

If $b=0$ then system $\rightarrow$ (marginal Stable)

when system is m.s. the f.o.o. are given by even power of s terms

$$f.o.o. = 8s^2+32=0$$

$$s = \pm j2$$

$$\boxed{\omega = 2 \text{ rad/sec}}$$

* CE $\Rightarrow$ as^3+bs^2+cs+d.

s^3	a	c
s^2	b	d
s	$\frac{bc-ad}{b}$	0
s^0	$d>0$.	

System $\rightarrow$ Stable

(1) Cond. $d>0$

(2) $\frac{bc-ad}{b} > 0$.

$bc > ad$.

$bc < ad$ (us)

$bc = ad$ (ms.)

Q. Find the system stability to following chao. equation.

CE → $s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$.

s^0 coeffecient shift diag By leaving one row before one colomn

s^4	1	3	5
s^3	2	4	0
s^2	1	5	
s	-6	0	
s^0	5		

2 Sign changes
unstable
2 Roots RH s-plane
2 Roots LH s-plane.

CE → $s^4 + 2s^3 + 3s^2 + 2s + 1 = 0$.

s^4	1	3	1
s^3	2	2	
s^2	2	1	
s^1	1		
s^0	1		

No sign change
(Stable)
4-poles LH s-plane.

CE - $s^4 + 2s^3$.

s^4	1	3	2
s^3	2	1	
s^2	2.5	2	
s	-0.66		
s^0	2		

2 Sign changes
unstable

CE - $s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$.

s^4	1	2	8
s^3	2	4	
s^2	$0 = \varepsilon$	8	
s	$\frac{4\varepsilon - 16}{\varepsilon} = 4 - \frac{16}{\varepsilon} = -\infty$		
s^0	8		

* whenever any one element is zero in first colomn replaced zero by smallest positive constant ε and continue the Routh stability. finally substitute $\varepsilon = 0$ and check the no. of sign changes.

(*) $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

s^5	1	2	3
s^4	1	2	15
s^3	$0 = \varepsilon$	-12	0
s^2	$\frac{2\varepsilon + 12}{\varepsilon}$ $+K$	15	
s	$\frac{-12K - 15\varepsilon}{K}$		
s^0	15		

∴ two sign changes. (unstable)

two Roots · RH Spl

three Roots LH (Spla

(*) $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0.$

s^5	1	3	2
s^4	$1 s^4$	$3 \cdot s^2$	$2 s^0$
s^3	~~0~~ 4	~~0~~ 6	~~0~~
s^2	3/2		
s^1	2/3		
s^0	2		

—— (Power × coefficient).

entire Row zero ; we required to form Auxilary equation.

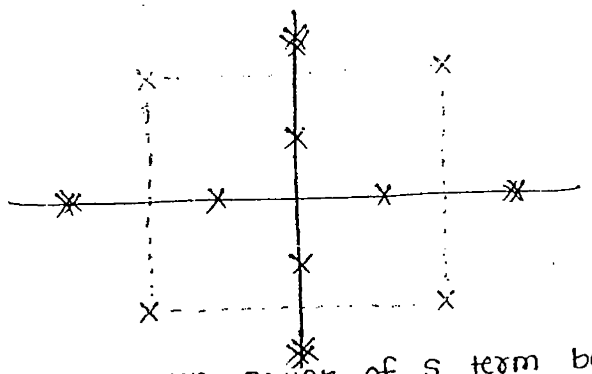
$$AE = s^4 + 3s^2 + 2$$

$$\frac{d(AE)}{ds} = 4s^3 + 6s.$$

Procedure -

whenever in Routh tabular form the entire Row become the zero then we require to form the Auxilary equation and differentiate the Auxilary equation and Replace the zeroes by co-effic of differential Auxilary Equation.

* in the Routh tabular form the Row of zero occurs only when the poles are located Symmetrical about the origin.

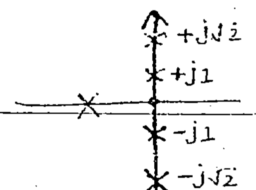


* The A·E. Should consist only even power of s term because Roots of A·E. are Symmetrical about the Real axis. the Roots of A·E. are nothing but Closed loop poles.
* The Row of zeroes occurs only odd Power of s power Rows.

$$AE = s^4 + 3s^2 + 2 = 0$$

$$(s^2+1)(s^2+2) = 0$$

$s = \pm j$ $\quad$ $s = \pm\sqrt{2}j$

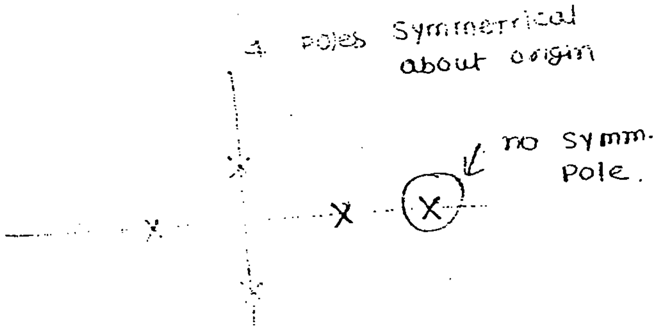


whenever in Routh tabular form only once the Row of zero occur and all the Coefficients in 1st coloms are +ve then system is m.s. because the poles on Imagenary axis which are non-repeated.
There is no chance that poles lies in R-H S-Plane.

+
+
+
+

IMP.
* The sign change occurs below the A.E. there must be a Symmetrical pole in the left side to pole placed in the right whereas. the sign change occurs above the A.E.'s there is no symmetrical pole placed in the left side to the pole placed in the right side.

−
+
+
+
+
−



$$s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$$

s^6	1	4	5	0 2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	~~0~~ 8	~~0~~ 8	0 0	0
s^2	2	2	0	
s^1	~~0~~ 4	0 0		
s^0	2			

two times entire Row becomes zero
consider only ① A.E. not ⑪

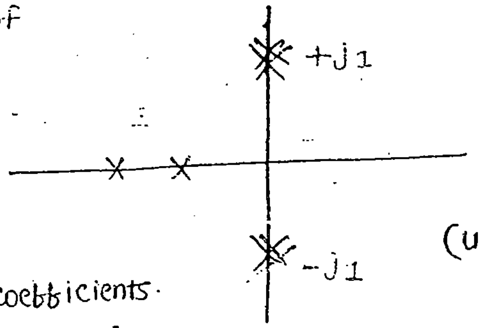
⑪ ⑪ ⑫ shows Repetitive nature of pole.

$$A.E. = 2s^4 + 4s^2 + 2 = 0$$

$$s^4 + 2s^2 + 1 = 0$$

$$(s^2+1)^2 = 0$$

** The [illegible] zero indicate no. of poles are repeated.



Note :-

whenever many times Rows are zero occurs in Routh tabular form and all the coefficients in the first colomn are +ve then the system is unstable. because the poles on imagenary axis which are repeated.

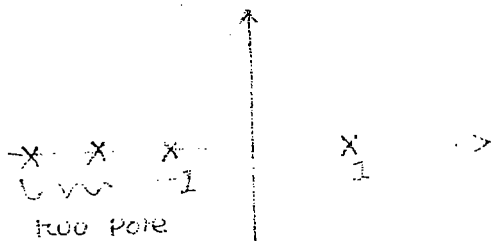
* $s^4 + s^3 - s - 1 = 0 \quad = \quad s^4 + s^3 + 0s^2 - s - 1 = 0$
the no. of poles in Right Half of S-plane are

s^4	1	0	-1
s^3	1	-1	
s^2	1	-1	
s	0	0	
s^0	-1		

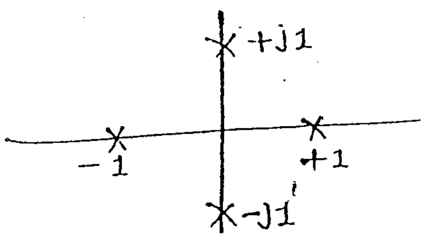
Row zero. (only two pole symm. about origin).

$A.E. = s^2 - 1 = 0$

$s = \pm 1$



* Identify the Routh tabular form to the given poles location in the S-plane.



* 1 time Row of zero
Because poles are symm. but not repeated

* 1 Sign change below A.E. because Symm. pole placed in the Right

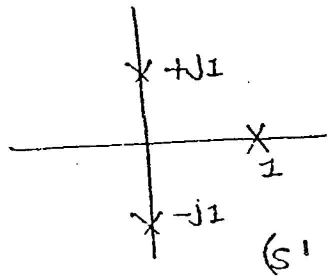
CE - $(s^2+1)(s^2-1) = 0$

$s^4 - 1 = 0.$

(*)

s^4	$1s^4$	$0s^2$	$-1s^0$
s^3	$\cancel{0}\ 4$	$\cancel{0}\ 0$	$\cancel{0}\ 0$
s^2	$\cancel{0}\ \varepsilon$	-1	
s^1	$4/\varepsilon$		
s^0	-1		

① Sign change.



$$(s^2+1)(s-1)=0$$

A.E. → $s^3 - s^2 + s - 1 = 0$

s^3	1	1
s^2	-1	-1
s	$\cancel{0}\ -2$	0
s^0	-1	

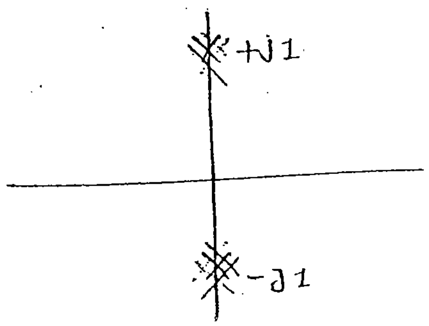
(s^1 Row must be zero)

1 sign change above A.E. because no symm. pole to the pole placed in the RH

Below auxilary eq. all are +ve

But above auxilary eq. ① sign change.

Q.



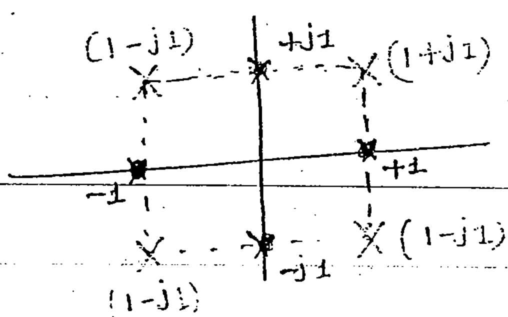
$$(s^2+1)^3 = 0$$

$$s^6 + 1 + 3s^2 + 3s^4 = 0$$

$$s^6 + 0s^5 + 3s^4 + 0s^3 + 3s^2 + 1 = 0$$

s^6	1	3	3	1
s^5	$\cancel{0}\ 6$	$\cancel{0}\ 12$	$\cancel{0}\ 6$	0
s^4	1	2	1	0
s^3	$\cancel{0}\ 4$	$\cancel{0}\ 4$	$\cancel{0}\ 0$	
s^2	1	1	0	
s^1	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	
s	1			

Q.



$$(s^2+1)(s^2-1)=0$$

$s^4 -$

CE $\quad [(S+1)^2+1]\,[(S-1)^2+1]=0$

$$(s^2+2s+2)\,(s^2-2s+2)=0$$

$$s^4+4=0$$

s^4	1	0	4
s^3	0	0	0 → (1 Row)
s^2	$0=\varepsilon$	4	
s	$-16/\varepsilon$		
s^0	4		

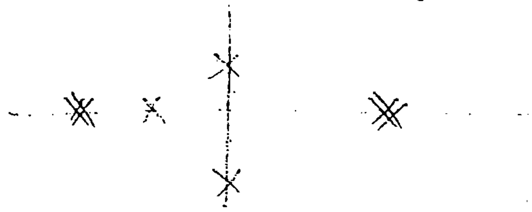
(*) Identify the no. of poles in the left, Right or the Img. to the given sampled Routh tabular form.

s^7	+			
s^6	+			
s^5	+ 0	0	0 →	
s^4	+			
s^3	+ 0	0	0 →	
s^2	−			
s^1	−			
s^0	+			

order of A.E. = 6^{th} means 6 pole symm. about origin.

two sign changes below A.E. i.e. 2 poles RH of S-plane and symm. poles left half S-plane.

2 times Row = 0. means 2 poles are repeated which symm. about origin. (It may be at real or Img. axis).



(*)

s^7	−			
s^6	+			
s^5	+ 0	0	0 →	
s^4	+			
s^3	+			
s^2	+			
s	+			
s^0	−			

order of A.E. = 6^{th}

1 Row zero → non repeated poles But symm.

1 sign change − below A.E. Symm. pole in the left Half to ~~the poles placed in Right Half~~

$s^3 + 8s^2 + 4s + K = 0$. $\underbrace{\quad}_{32}$ $(0<K<32)$ Value for system Stability.

s^3	1	4
s^2	8	K
s	$\frac{32-K}{8}$	0
s^0	K	

Stable $\boxed{K>0}$

$\frac{32-K}{8} > 0$

$32 - K > 0$

$-K > -32$

$\boxed{K < 32}$

* For Stability $\rightarrow \boxed{0<K<32}$

(ii) Find the K value to become the system m.s. for the given sys. equa

(iii) Find the f.o.o. when system is m.s. to then

* For m.s. never consider s^0 coefficient $= 0$. then which is otherwise A.E. consist odd power not possible (undesirable).

entire Row Become zero

For marginal stability

$\frac{32-K}{8} = 0$

* $\boxed{K_{marginal} = 32}$

$8s^2 + K = 0$

$8s^2 + 32 = 0$

$8s^2 = -32$

$s^2 = -4$

$s = \pm j2$

$\boxed{\omega = 2 \text{ rad/sec}}$

Q. $2s^3 + 10s^2 + 5s + (K+5) = 0$.

s^3	2	5
s^2	10	K+5
s	$\frac{2K-40}{10}$	0
s^0	K+5	

$K+5>0$

$\boxed{K>-5}$

$50 > (2K+10)$

$\boxed{K<20}$

$\boxed{-5<K<20}$

$2K+10 = 50$

$\boxed{K_{mar.} = 20}$

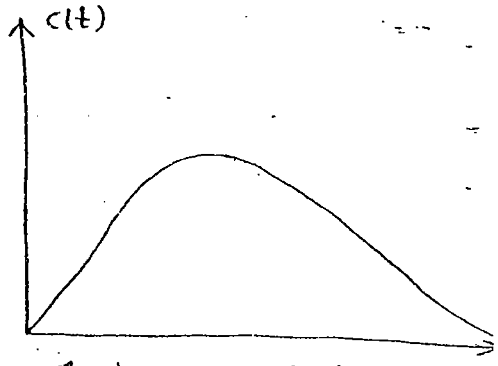
$10s^2 + 25 = 0$

$s^2 = -25/10$

$s = \pm j\,5/\sqrt{}$

I.L.T.

$$c(t) = \omega_n^2 t\, e^{-\omega_n t}$$

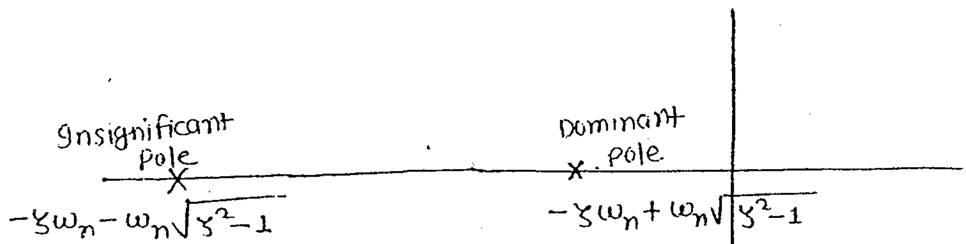


1. damped oscillation.

* when $\zeta = 1$ Both the poles on –ve Real axis at the same location. the system is stable System Response is called critical damped system because the Sys. Response critically generates one damped oscillation

* $\zeta > 1$ (overdamped System) →

$$S_1, S_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$



$$T = \frac{1}{\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}}$$

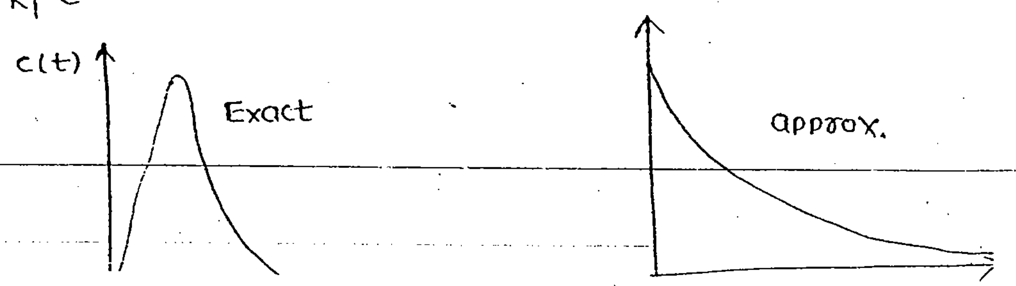
f.o.o. = 0 rad/sec

System = Stable

$$C(S) = \frac{\omega_n^2}{(\underbrace{S + \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}}_{a})(\underbrace{S + \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}}_{b})}$$

$$= \frac{K_1}{\underbrace{S + \zeta\omega_n - \omega_n\sqrt{\zeta^2-1}}_{a}} - \frac{K_2}{\underline{S + \zeta\omega_n + \omega_n\sqrt{\zeta^2-1}}}$$

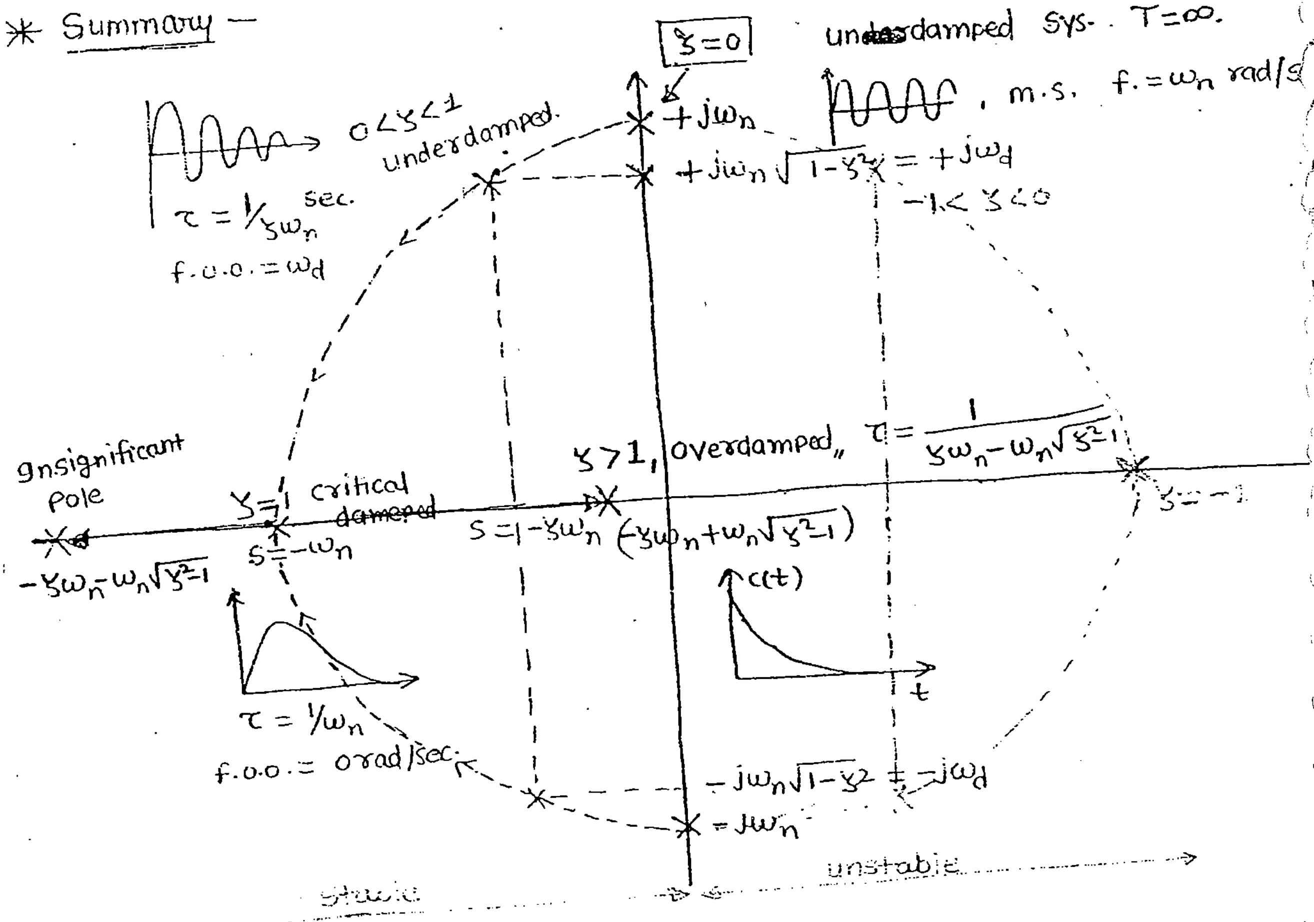
$$c(t) = K_1 e^{-(\zeta\omega_n - \omega_n\sqrt{\zeta^2-1})t} - K_2 e^{-(\zeta\omega_n + \omega_n\sqrt{\zeta^2-1})}$$



$\zeta > 1$ poles on $-ve$ Real axis

different locⁿ System is stable. the System. Response is overdamped System. because the System. Response eliminates or overcome the damped oscillations.

$*$ Summary —



Conclusion

$*$ as ζ increases from 0 to 1, the poles moves to left and near to the real axis in this case

① the Time Constant is decreases, Hence Settling time also decreases.

② damped osc. ω_d are decreases, as $\omega_d \downarrow\downarrow$ the time Specification (t_d, t_r, t_p) must be increases.

③ Relative Stability must be Improved.

$*$ as ζ increases from 1 to ∞, one of the poles move towards the origin on the Real axis in this case

① Time Const. $\downarrow\downarrow$

② Settling time $\uparrow\uparrow$

③ $\omega_d = 0$ (no. osc.)

Q.

$$GH = \frac{K}{S(S+2)(S+4)(S+6)}$$

$1 + GH = 0$

$S(S+2)(S+4)(S+6) + K = 0$

$(S^2+2S)(S^2+10S+24) + K = 0$

CE = $S^4 + 12S^3 + 44S^2 + 48S + K = 0.$

{Product of S terms} + {Σ of all constants} + {Σ of Product of all possible combination of any 2 constant} + {Σ of all the possible " " of any 3 constant}.

$(S+1)(S+2) = S^2 + 3S + 2$

$(S+1)\underbrace{(S+2)}_{2}\underbrace{(S+3)}_{6} = S^3 + 6S^2 + 11S + 6.$

S^4	1	44	K
S^3	12	48	
S^2	40	K	
S	$\frac{160-K}{40}$		
S^0	K		

Stability

$K > 0$

$\frac{160-K}{40} > 0$

$\boxed{0 < K < 160}$

for m.s.

$\frac{160-K}{40} = 0$

$\boxed{K = 160}$

A.E.

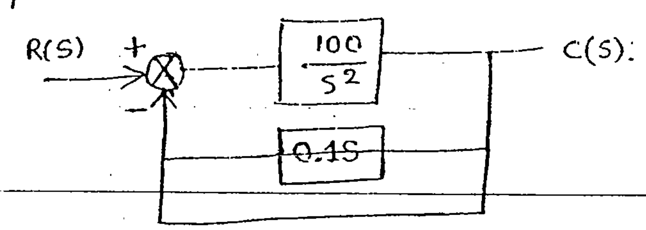
$40S^2 + K = 0$

$40S^2 + 160 = 0$

$S = \pm j2$

$\boxed{\omega = 2 \text{ rad/sec.}}$

* Check the Stability



$$\frac{C(s)}{R(s)} = \frac{100}{s^2+10s+100.}$$

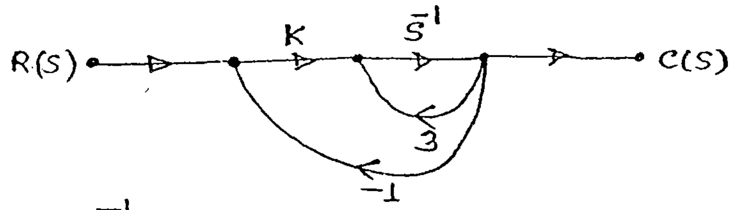
$$\left.\begin{array}{lll} s^2 & 1 & 100 \\ s & 10 & \\ s^0 & 100 & \end{array}\right\} \text{— Stable}$$

(*) The system shown in figure remains stable.

① $K < -1$ ③ $1 < K < 3$.

② $-1 < K < 1$ ④ $K > 3$.

Soln



$$\frac{C(s)}{R(s)} = \frac{K s^{-1}}{1 - 3s^{-1} + K s^{-1}}$$

$$= \frac{K}{s - 3 + K}$$

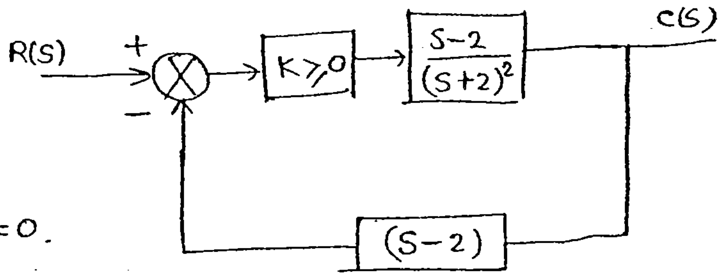
$$\begin{array}{ll} s^1 & 1 \\ s^0 & K-3 \end{array}$$

$$\boxed{K > 3}$$

(*) The f/b control system shown in figure is stable

ⓐ for all $K \geq 0$. ⓑ only if $K \geq 1$.

ⓒ only if $0 \leq K \leq 1$ ⓓ $0 \leq K \leq 1$



$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+2)^2 + K(s-2)^2}$$

$K(s-2) + s^2 + 4s + 4 + K(s-2)^2 = 0.$

$K(s-2) + s^2 + 4s + 4 + Ks^2 - 4Ks - 4K = 0$

$s^2(1+K) + s(4-4K) \cancel{+4-4K=0.}$
$+Ks - 2K$

$$s^2(1+K)+s(4-4K)+(4+4K)=0.$$

s^2 $K+1>0$ $4+4K$ $K>-1$ (not possible)

s $4-4K>0$ $\boxed{K<1}$

s^0 $4+4K>0$

$K>-1$

(d) $\boxed{0\le K<1}$ Stable.

* loop gain

$$GH = \frac{K}{s(s+2)(s+4)}$$

Value of K for which the sys. just become unstable is.

means marginal Stable.

$$s(s+2)(s+4)+K=0$$

$$s^3+6s^2+8s+K=0$$

$$48 \qquad K$$

$\boxed{K_{max.}=48}$

* A System Has $G(s) = \dfrac{K}{s^3+4s^2+8s}$.

For what value of K system produce continuous oscillatic (m.s.)

$$s^3+4s^2+8s+K=0$$

* Relative Stability → $\boxed{K=32}$

* A sys. has $G(s) = \dfrac{2}{s(s+1)(s+2)}$ $H(s)=1$

For what R-H criteria determine sys Relative stability about the line $s=-1$.

(R.S. is valid for only closed loop Stable System)

OR

check either the time constant greater, lesser or equal to 1 sec.

Soln →

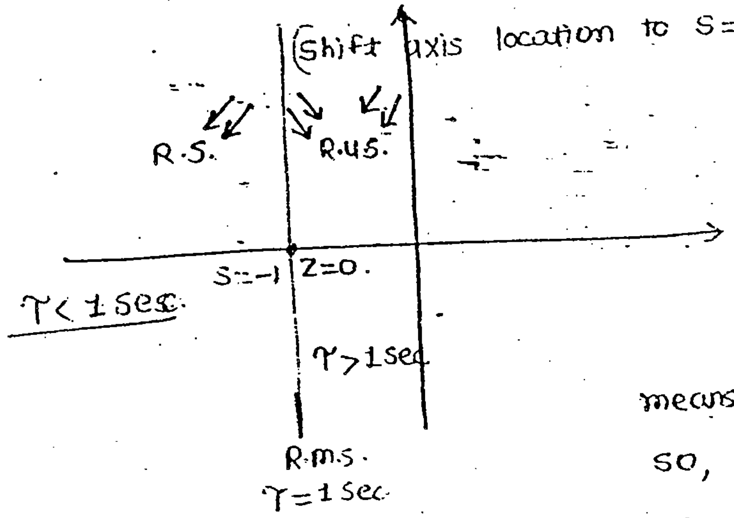
$$s(s^2+3s+2)+2=0$$

$$s^3+3s^2+2s+2=0$$

(internal > ext.)

s^3	1	2
s^2	3	2
s^1	4/3	0
s^0	2	

(Stable)



$z = 0$

means. $S+1 = 0.$

so, $S+1 = z$

$\boxed{S = z-1}$

$$(z-1)^3 + 3(z-1)^2 + 2(z-1) + 2 = 0.$$

* $G(S) = \dfrac{2}{(z-1)(z)(z+1)} + 2 = 0.$

CE = $z^3 - z + 2 = 0.$

z^3	1	-1
z^2	$\cancel{0}\varepsilon$	2
z	$\dfrac{-\varepsilon-2}{\varepsilon}$	0
z^0	2	

R.U.S.

2 sign changes.

2 poles b/w 0 & -

1 pole must be L-H of S plan, $S = -1$

* * $\boxed{S = z + \text{Axis shift location } (= -1/\tau)}$

** Note- The R-H criteria is not applicable to the exponential, the sine, cosine terms. because It gives the Infinite Series but the R-H criteria is applicable for finite no. of terms only.

* By using RH criteria we can get the approx. solⁿ to the exponential terms.

Q. Check the Stability.

$$G(S) = \frac{e^{-ST}}{S(S+1)}$$

$$= \frac{(1 - ST)}{S(S+1)}$$ (Neglect Higher order term).

CE ⇒ $S^2 + S + 1 - ST = 0.$

$S^2 + S(1-T) + 1 = 0.$

S^2	1	1
S	$1-T$	0
S^0		

$1 - T > 0$

$\boxed{T < 1 \text{ Sec.}}$ ** **

** ROOT LOCUS →

Purpose →

* To Find the Close loop System Stability
* To bind the Range of K value bor System Stability.
* To bind the $K_{marg.}$ and $\omega_{marg.}$ Value.
* Find the K Values for Undamped, Underdamped, critical damped and overdamped System.
* To bind the Relative Stability If the Root locus Branches moving towards the left then the System is Relatively Stable.
* If Root locus Branches moving towards the Right then System is Relatively unstable.

Definition —

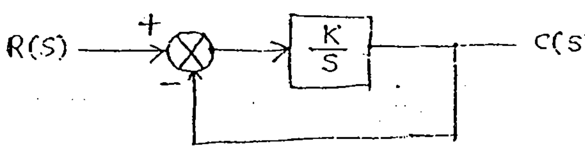
Root – Roots of char. equation

= closed loop poles.

Locus – Path

Root locus – closed loop pole path by varying K from 0 to

* Draw the Root locus – Root locus diag. is nothing But Closed l



IIT → Poles path.

the closed loop poles path ~~given by~~

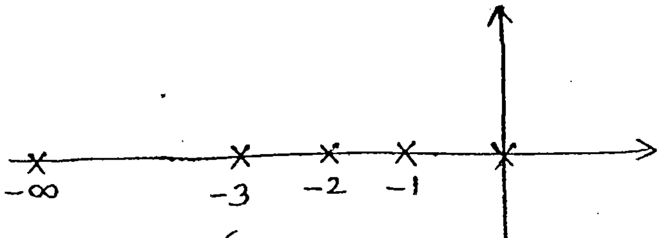
char. equation.

CE $\Rightarrow$ $1+G(S)=0$

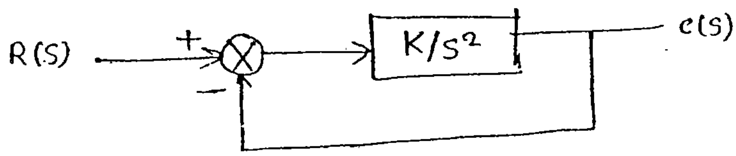
$$1+K/s=0$$

$$\boxed{S=-K}$$

K	Pole location $S=-K$
0	0
1	−1
2	−2
3	−3
⋮	⋮
∞	∞.



Q.

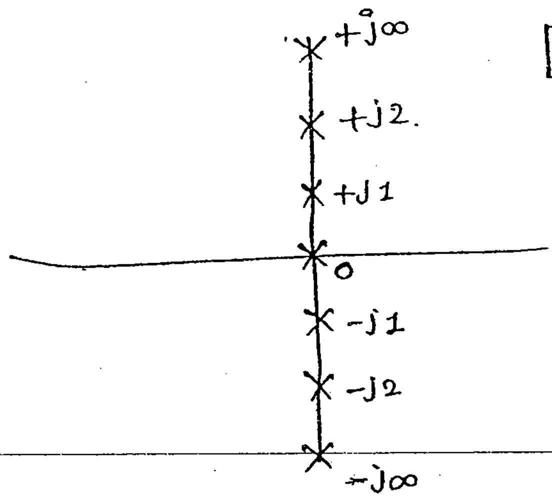


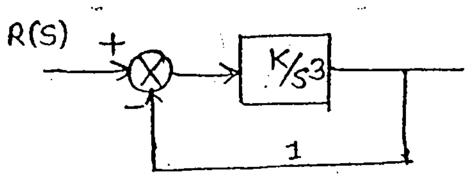
$$1+K/s^2=0$$

$$s^2+K=0$$

$$\boxed{S=\pm j\sqrt{K}}$$

K	Pole locn.
0	origin
1	$\pm j1$
10	$\pm j\sqrt{10}$
∞	$\pm j\infty$



* 

$s^3 + K = 0.$ (third order with unknown Paramete

as order increases finding the Roots for char. equation is very dibbicult Hence we can not draw the Root locus diagram by using char. equation.

* to draw the Root locus diagram we use the O.L.T.F.

→ Relationship b/w OLTF, CLTF, Poles and zero's :-

The OLTF

$$G(S)\,H(S) = K \cdot \frac{N(S)}{D(S)}$$

open loop poles → $D(S) = 0.$

open loop zero · $N(S) = 0$

* closed loop poles are given by char. equation.

$$1 + G(S)\,H(S) = 0.$$

$$1 + K\,\frac{N(S)}{D(S)} = 0$$

$D(S)$. → open Loop pole
$N(S)$ → open Loop zero

C.E. $D(s) + K\,N(S) = 0$ (which gives closed loop poles

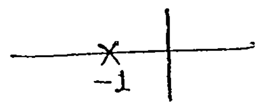
* closed loop poles are nothing but sum of open loop poles open loop zero with the function of system gain K.

case I $K = 0$ when

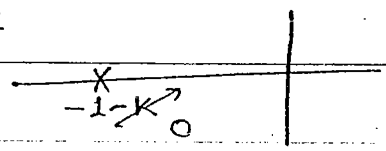
$$K = \left| \frac{-D(S)}{N(S)} \right|$$

closed loop pole $D(S) = 0.$

** $\boxed{\text{when } K=0 \qquad \text{CL Pole} = \text{OL pole}}$

$$G(S) = \frac{K}{S+1}$$ 

$$CLTF = \frac{K}{S+1+K}$$



Case-II → $K = \infty$.

CL poles $N(S) = 0$

when $K = \infty$ CL-pole = O.L. zero.

* Root locus diagram should be start at open loop poles where $K = 0$ and the Root locus diag. end at open loop zeros. where $K = \infty$.

→ Find where the Root locus diagram starts and end for

$$G(S)\,H(S) = \frac{K(S+1)}{S(S+2)(S+5)}$$

Solⁿ → Start at open loop Poles · $S = 0, -2, -5$.
end at open loop zero · $S = -1, \infty, \infty$.

For one pole there have one zero.

* to draw a Root locus diagram we required, no. of poles = no. of zero's

Because the Root locus diag. start at poles, end at zeros. i.e. that means no. of Poles must be equal to no. of zero.

If zero's are less assumed zero's at ∞, the direction of ∞ given by angle of asymptotes.

* Angle and magnitude condition →

The closed loop System stability given by char. equation.

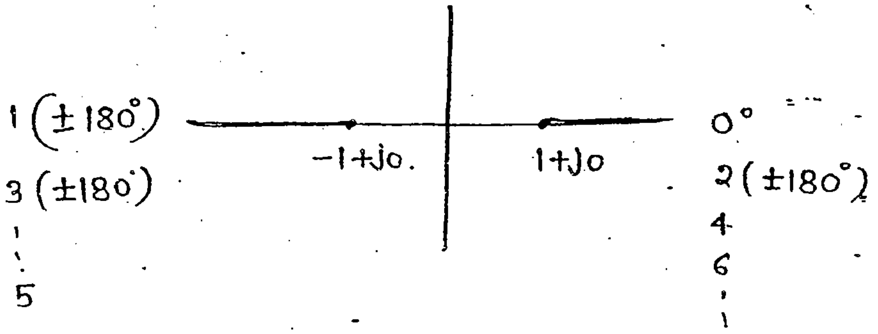
−ve F/b / DRL (Direct Root Locus) / 180° Rules	+ve f/b / IRL/CRL / 0°
CE: $1 + G(S)\,H(S) = 0$	$1 - G(S)\,H(S) = 0$
$G(S)\,H(S) = -1 + j0$	$G(S)\,H(S) = 1 + j0$.

The above equation gives the magnitude cond. and angle condition.

Angle condition →

$\angle G(S)\,H(S) = \angle(-1 + j0)$

$\angle G(S)\,H(S) = \angle(1 + j0)$



$= \underset{\sim}{\text{odd}}$ multiples of $\pm 180°$

$= \pm(2q+1)\,180°$

$q = 0, 1, 2.$

$\underset{\sim}{\text{Even}}$ multiples of $\pm$

$\pm(2q)\,180°$

$q = 0, 1, 2$

Rule.	Direct Root locus.	Inverse/ Complementary Root locus.
R-3	ODD	Replaced by → EVEN.
R-4	2q+1	→ 2q.
R-6 (case-III)	LEFT-most	→ RIGHTmost
R-8	180°	→ 0°

Purpose of angle condition -

to check the any point lies on the root locus or not that means all the points on Root Locus must Satisfy the angle conditions.

* Check whether the following points lies on Root locus or not

$$G(s)H(s) = \frac{K}{s(s+5)(s+10)}$$

① $s = -3$ ② $s = -6$

$$\angle G(s)H(s)\Big|_{s=-3} = \frac{\angle K}{\angle \underset{-3}{s}\ \angle(\underset{-3}{s}+5)\ \angle(\underset{-3}{s}+10)}$$

$$= \frac{\angle K}{\angle -3\ \angle 2\ \angle 7} = \frac{\angle 0°}{\angle \pm 180°\ \angle 0 \angle 0.}$$

$$= 1(\mp 180°)$$

Satisfies angle cond.

$$\angle G(s)H(s)\Big|_{s=-6} = \frac{\angle K}{\angle s \ \angle(s+5)\ \angle(s+10)}$$

$$= \frac{\angle K}{\angle -6 \ \angle -1 \ \angle 4}$$

$$= \frac{0}{(\pm 180^\circ)(\pm 180^\circ)(0^\circ)}$$

$$= 2\,(\mp 180^\circ)$$

$$= 2 \times (\mp 180)$$

= even multiple of 180°
(Not satisfies A.C. So given point not on Root locus)

* magnitude condition →

$$|G(s)H(s)| = 1$$

at any point which is on Root locus.

* The mag. condition is valid when given point is on Root Locus only.
that means Angle condition must be satisfied to apply magnitude condition.

Purpose →
To find the system gain at any point which is on Root locus

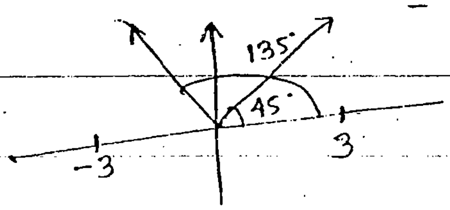
Problems →
* Find the system gain at the point $s = -3 + j3$ to the given system $G(s)H(s) = \frac{K}{s(s+6)}$

$$\angle G(s)H(s)\Big|_{s=-3+j3} = \frac{\angle K}{\angle s \ \angle(s+6)}$$

$$= \frac{\angle K}{\angle(-3+j3)\ (\angle(3+j3))}$$

$$= \frac{0^\circ}{\angle 135^\circ \ \angle 45^\circ} = 1\angle($$

Never take the angle w.r.t. 180° always take w.r.t. 0°



Satisfies A.C. given point on Root locus.

$$\left| \frac{K}{s(s+6)} \right|_{s=-3+j3} = 1$$

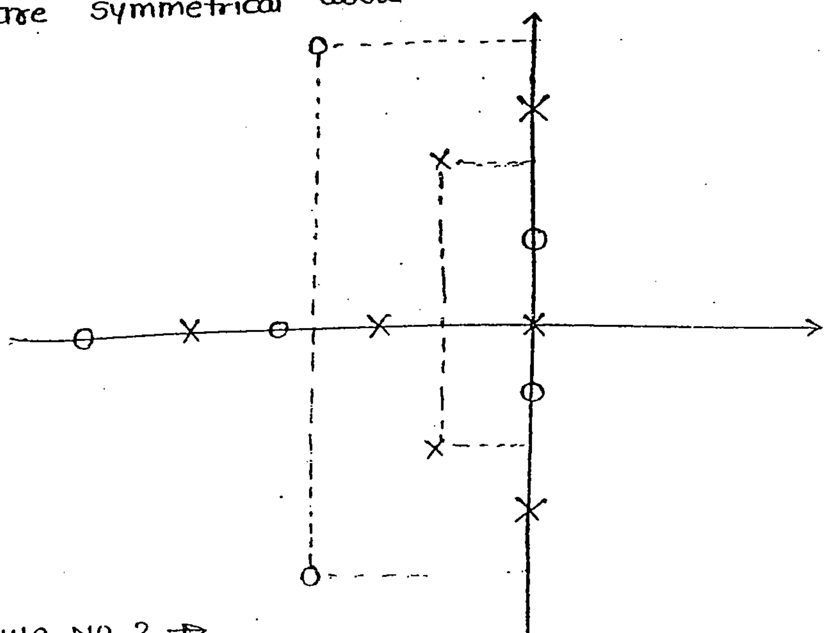
$$\frac{K}{\sqrt{18}\cdot\sqrt{18}} = 1$$

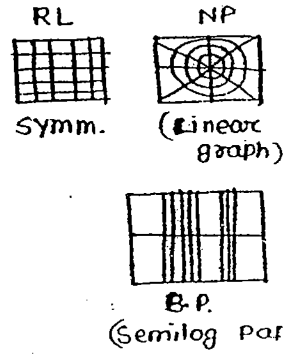
$$\boxed{K = 18}$$

Construction Rules of Root locus -

Rule No. 1 →

(Symmetrical) → A Root locus diagram is symmetrical about the Real axis because locn of poles and zero's in the S-P are symmetrical about the Real axis.





Rule No. 2 →

Number of Loci - (No. of Root locus Branches)

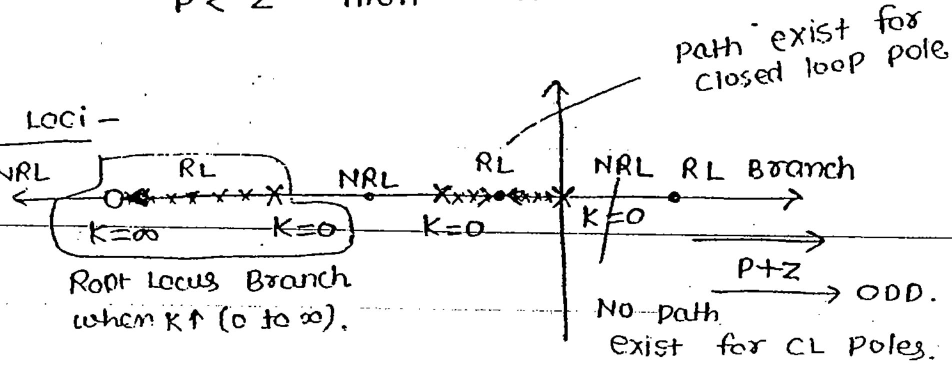
No. of Root locus Branches completely depends on no. of Poles a zero's

case-① → If no. of poles > no. of zero then no. of Root locus Branches N = P.

P > Z then N = P.
P < Z then N = Z.

* **Rule-3** →

Real Axis Loci -

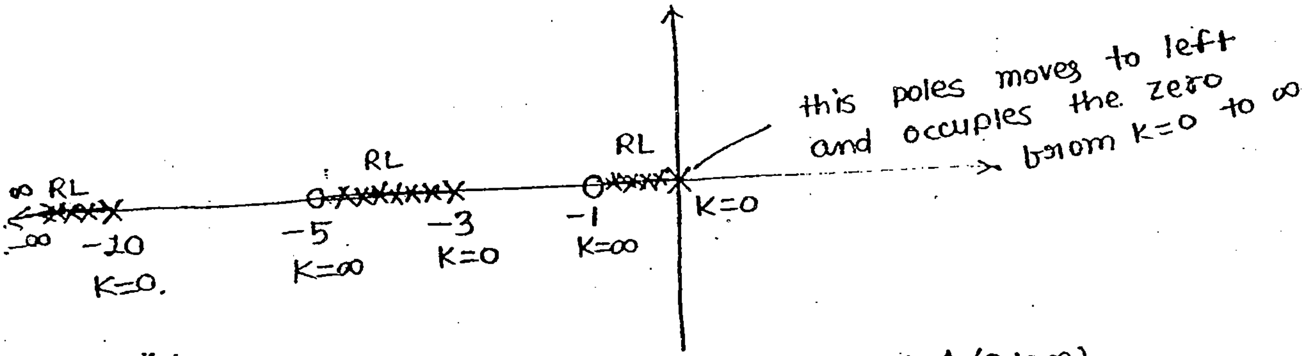


* A Point to exist on Real axis ... the Right Hand side of that point should be odd. of the poles and zeros to the point should be odd.

* The poles must move only on Root locus Branch once the pole reach the zero then It becomes the complete Root locus Branch for that particular pole.

* Find The section of Real axis which Belongs to Root locus.

$$G(S)\,H(S) = \frac{K(S+1)(S+5)}{S(S+3)(S+10)}$$

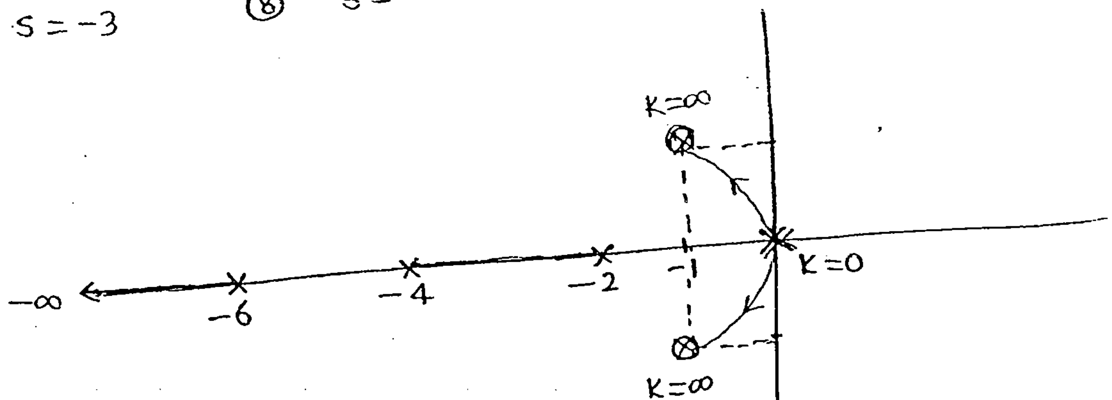


complete Root locus Diagram $K \uparrow (0 \text{ to } \infty)$
(All the Poles Reaches to the zeroes.)

* Identify the following points which are on Root Locus Branch.

$$G(S)\,H(S) = \frac{K(S^2+2S+2)}{S^2(S+2)(S+4)(S+6)}$$

① $S=0$
② $S=-1$
③ $S=-2$
④ $S=-3$
⑤ $S=-4$
⑥ $S=-5$
⑦ $S=-6$
⑧ $S=-\infty$
⑨ $S=-1+j1$
⑩ $S=-1-j1$



**
Note—
at the position of all the poles and zeroes there must be a Root Locus Branch because Root Locus ... start at poles, end at zero.

Q.

$$GH = \frac{K(s+10)}{s^2(s+1)}$$

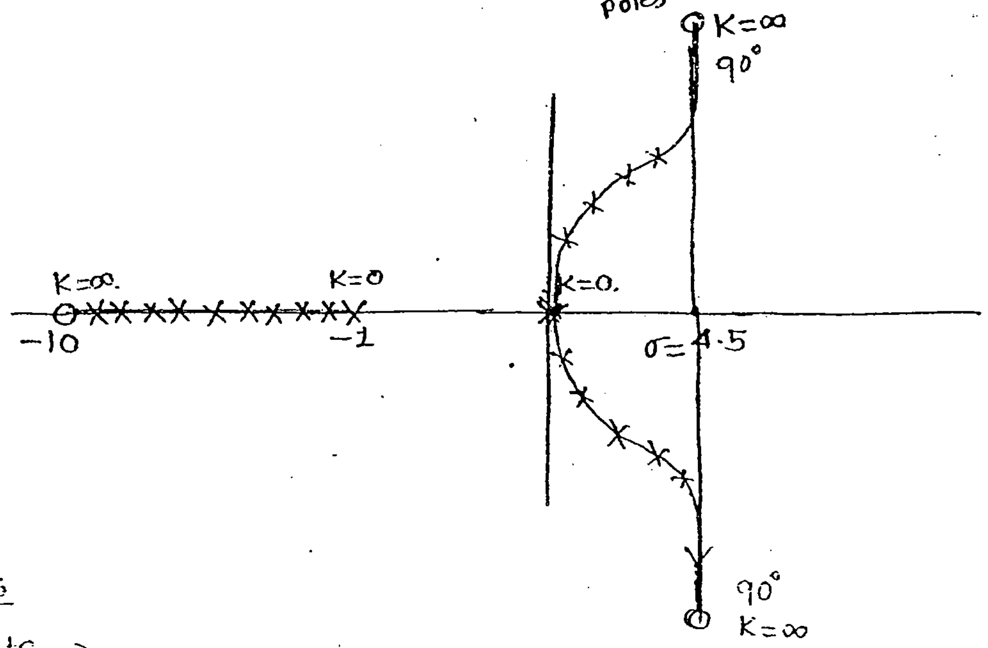
$$\theta_1 = \frac{(2\times 0+1)180^\circ}{3-1} = \frac{180^\circ}{2} = 90^\circ$$

$$\theta_2 = 270^\circ$$

$$\theta_3 = \cancel{450^\circ}$$

$$\sigma = \frac{-1+10}{2} = \frac{9}{2} = 4.5$$

Howmany poles are moving $\frac{\pm 180^\circ}{n}$



* Rule No.-6

* Break Points → two or more poles Directly locate or two are more poles meet at any particular location then it is called Break Point.

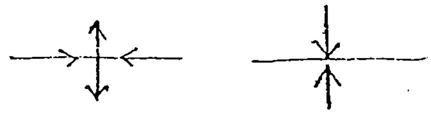
The point at which Root locus

Break away Point → The Point at which Root locus branches leaves the Real axis called Breakaway point.

Break in points → The point at which Root locus Branches enter into the Real axis then it is called Breakin Points

* The Root Locus Branches enter or leave the Real axis with an angle of $\boxed{(\pm 180^\circ)/n}$ where

n = no. of poles. meet at that Breakpoint.



Rule No. 4 –

Asymptotes – Asymptotes are the Root Locus Branches which are approch to the ∞ the

no. of Asymptotes $N = (P - Z)$

Angle of Asymptotes $\theta = \dfrac{(2q+1)\,180^\circ}{(P-Z)}$

where $q = 0, 1, 2, 3 \ldots$

→ The Asymptotes are Symmetrical about Real axis.
→ The Asymptotes gives the direction of zeros, when the poles are greater than zero $(P > Z)$.

Rule No. 5

Centroid – It is nothing but Intersection point of Asymp on the Real axis.

$$\sigma = \frac{\sum(\text{Real Part of Poles}) - \sum(\text{Real part of zeros})}{(P-Z)}$$

The centroid may be located anywhere on the Real axis. It may or may not be on Root locus Branch.

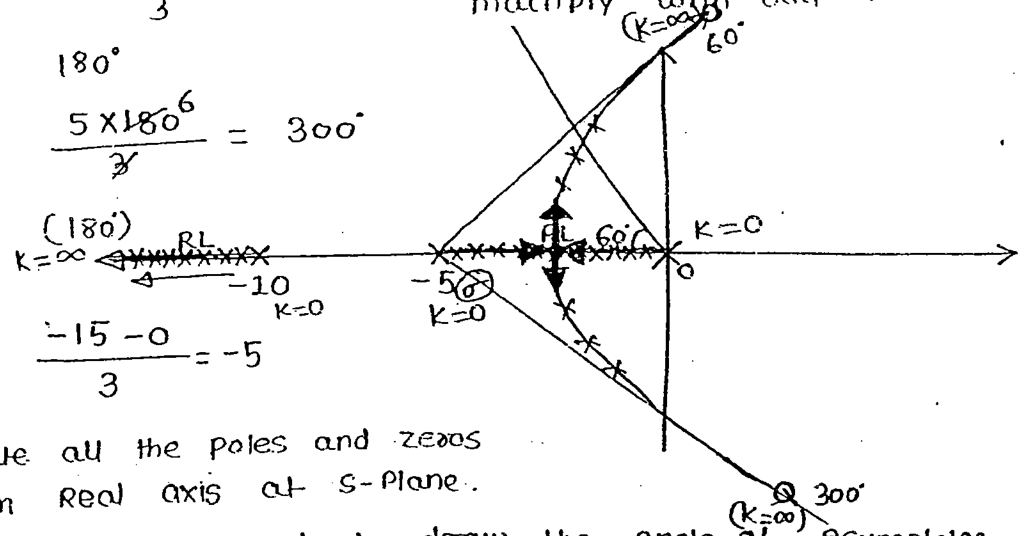
* Calculate the Angle of Asymptotes and centroid to the Following System

$$G(S)\,H(S) = \frac{K}{S\,(S+5)\,(S+10)}$$

$\theta_1 = \dfrac{(2 \times 0 + 1)\,180^\circ}{3} = 60^\circ$ (For successive angle we should multiply with odd number).

$\theta_2 = 180^\circ$

$\theta_3 = \dfrac{5 \times \cancel{180}^{6}0}{\cancel{3}} = 300^\circ$



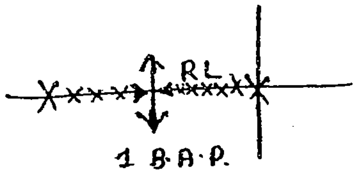
$\sigma = \dfrac{-15 - 0}{3} = -5$

Locate all the poles and zeros on Real axis at S-Plane.

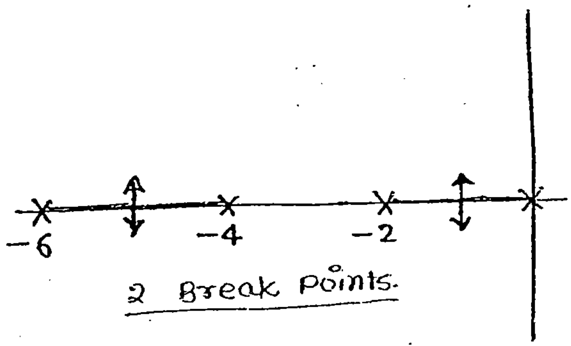
centroid is required to draw the angle of Asymptotes.

** Finding the existance of Break point →

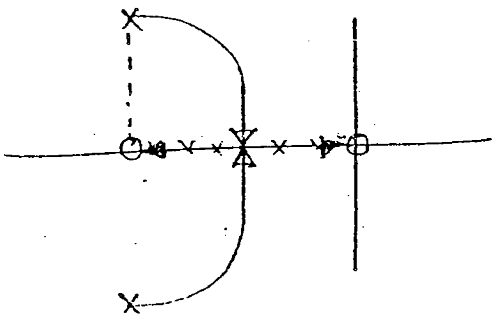
case-I → whenever any two poles are adjacently placed there exist a Root locus Branch then there should be in b/w there exist the minimum one Breakaway point in b/w adjacently placed poles



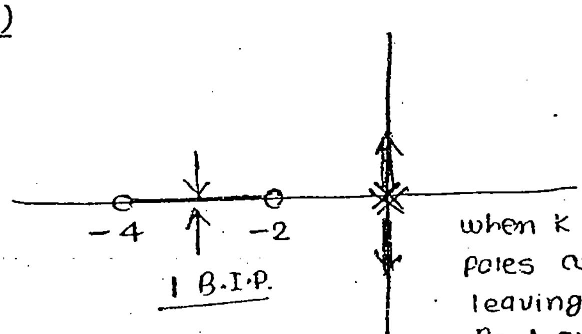
$$G(s)\,H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$



case-II — whenever there are two adjacent placed zeros are exist in b/w there exist the Root locus Branch then there should be the minimum one Break in point in b/w adjacently placed zero's



$$G(s)\,H(s) = \frac{K(s+2)(s+4)}{s^2}$$



* whenever there exist the left most side zero to the left most side of that zero there exist a Root locus Branch then there should min. one Break in point to the left most side of zero when

OR

* For a valid Breakpoint K value should be positive.

→ Find the location of Break point

① $G(S)H(S) = \dfrac{K}{S(S+2)}$

② $G(S)H(S) = \dfrac{K}{S(S+2)(S+4)}$

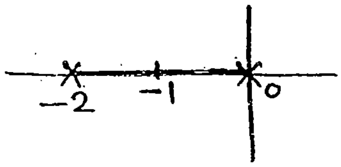
③ $G(S)H(S) = \dfrac{K(S+4)}{S(S+2)}$ Because of zero's Replace $G(S)H(S) = -1$

①

S^2+2S

$2S+2=0$

$\boxed{S=-1}$ (Valid)



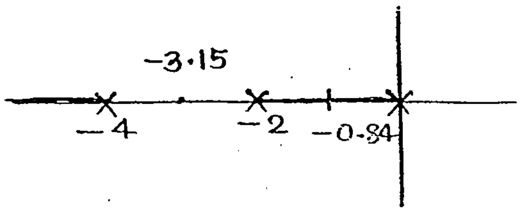
②

$S(S+2)+(S+4)(2S+2)=0$

$S^2+2S+2S^2+10S+8=0$

$3S^2+12S+8=0$

$S=-0.814,\ -3.15$



③

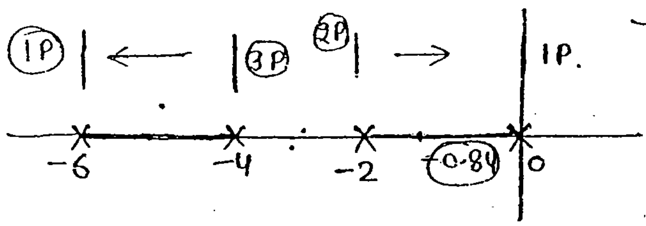
$$-1 = \frac{K(S+4)}{S(S+2)}$$

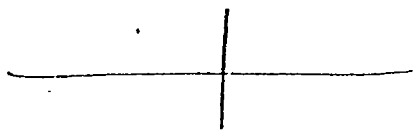
$$K = \frac{-S^2-2S}{S+4}$$

$$\frac{dK}{dS} = \frac{(S+4)(-2S-2)-(-S^2-2S)}{(S+4)^2} = 0$$

$$= -2S^2-8S-2S-8+S^2+2S=0$$

* $\boxed{S = -1.17,\ -6.82}$





Control System.

* Rule No. 7 →

Intersection point with Imagenary axis obtained by R-H Criteria

Procedure -

① Form the char. equation

② write the Routh tabular form.

~~G(S) H(S)~~

③ Find the K_{marg}.

④ Form the Auxilary equ^n. Roots of A·E. gives the valid and Invalid Intersection Point with Imagenary axis.

⑤ the valid Intersection Point is the one forwhich $K_{marg.} = +ve$,

* Find intersection point with Imagenary axis.

$$G(S)\, H(S) = \frac{K}{S \cdot (S+2)\,(S+4)}.$$

$$S(S^2+6S+8)+K=0$$

$$S^3+6S^2+8S+K=0$$

S^3	1	8
S^2	6	K
S	$\frac{48-K}{6}$	
S^0	K	

for $K_{marg.}$ $\frac{48-K}{6} = 0.$

$$\boxed{K=48}$$

$$6S^2+48=0$$

$$\boxed{S = \pm j2\sqrt{2}}$$

∵ K = +ve so valid Intersection point.

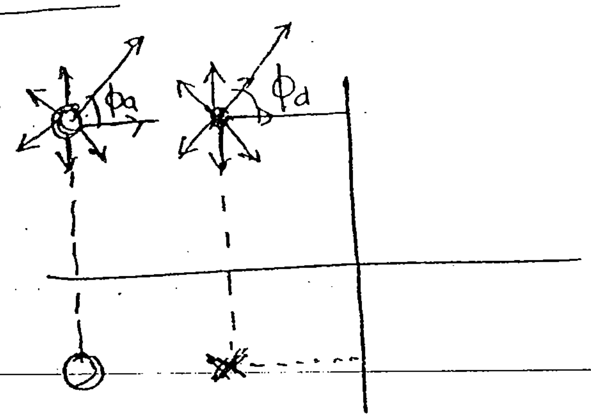
* Rule No. 8

Angle of Departure and Angle of Arrival —

Angle of Departure is calculated at complex conjugate pole.

* Angle of Departure is calculated at complex conjugate pole whereas then angle of arrival is calculated at complex conjugate zero.

at what angle pole terminate (arrive). given by angle ~~of~~ arrival.



Angle of Departure –

* It gives the with what angle the pole leaves (or Depart) from the initial position given by Angle of departure

$$\phi_d = 180^\circ + \angle GH \Big|_{\text{at a +ve Imagenary Complex pole.}}$$

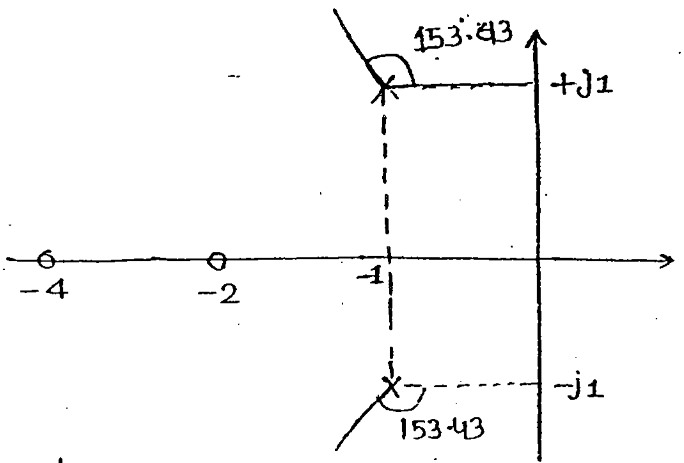
Angle of arrival –

* It gives that with what angle the pole terminates or end near the complex zero

$$\phi_a = 180^\circ - \angle GH \Big|_{\text{at a +ve Imagenary complex zero.}}$$

→ calculate Angle of Departure, at a complex conjugate Poles for

$$G(s)H(s) = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$$



mirror Image

$$\angle G(s)H(s)\Big|_{s=-1+j1} = \frac{\angle K \; \angle(s+2) \; \angle(s+4)}{\angle(s+1+j1) \; \angle(s+1-j1)}$$

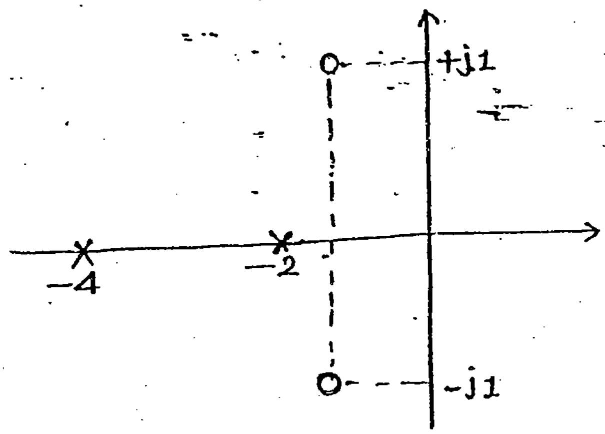
$$= \frac{\angle K \; \angle(1+j1) \; \angle(3+j1)}{\angle 0 \; \angle j2}$$

$$= \frac{\angle 0 \cdot \angle 45 \; \angle 18.43}{\angle 0^\circ \; \angle 90^\circ} = -26.58^\circ$$

$$\phi_d = 180^\circ + \angle G(s)H(s) = 180^\circ - 26.58^\circ = \underline{153.43^\circ}$$

→ calculate angle of arrival

$$G(s)H(s) = \frac{K(s^2+2s+2)}{(s+2)(s+4)}$$



$$\angle G(S)H(S) = \frac{K \angle (S+1+j1) \ \angle (S+1-j1)}{}$$

$\angle$ of departure = $\angle$ of arrival.

* whenever all the poles and zero are interchange then $\boxed{\phi_d = \phi_a}$
* Breakin point = Breakaway point

The shape of the Root locus diagram is same except the direction.

* Draw the Root Locus diagram to the following T.F.

① $G(S)\,H(S) = \dfrac{K}{S(S+2)}$

② $G(S)\,H(S) = \dfrac{K}{S(S^2+2S+2)}$

③ $G(S)\,H(S) = \dfrac{KS}{S^2+4}$

④ $G(S)\,H(S) = \dfrac{K(S+2)(S+4)}{(S^2+2S+2)}$

⑤ $G(S)\,H(S) = \dfrac{K(S^2+2S+2)}{(S+2)(S+4)}$

⑥ $G(S)\,H(S) = K/S, \ K/S^2, \ K/S^3, \ K/S^4$

⑦ $G(S)\,H(S) = \dfrac{K}{S(S+1)^2(S+2)}$

⑧ $G(S)\,H(S) = \dfrac{K(S+1)^2}{S(S+2)}$

⑨ $G(S)\,H(S) = \dfrac{K}{S(S^2+2S+2)(S+4)}$

$\dfrac{K}{\ldots}$

(11) $G(S)\,H(S) = \dfrac{K}{S(S+2)(S^2+2S+2)}$

(12) $G(S)\,H(S) = \dfrac{K}{S(S+2)(S^2+4S+5)}$

(13) $G(S)\,H(S) = \dfrac{K}{S(S+4)(S^2+4S+20)}$

(14) $G(S)\,H(S) = \dfrac{K(S+1)}{S(S^2+6S+10)}$

(15) $G(S)\,H(S) = \dfrac{K(S+1)}{S^2-2S+2}$

(1) Locate the poles in the S-plane and identify the Real axis Root locus Branches and Breakpoint.

(2) Find the angle of Asymptotes and centroid. If required Find the angle of departure and arrival.

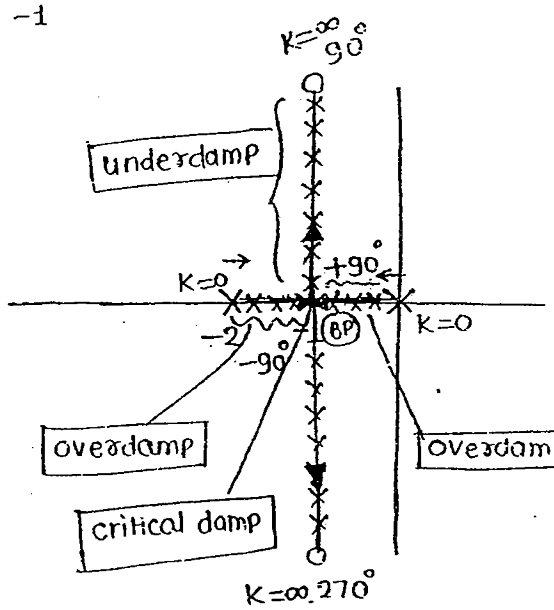
(3) where is K value from 0 to ∞, Identify the path from pole to zero such that the Root locus diagram is symmetr about Real axis and all the poles should Reaches the zeros

$G(S)\,H(S) = \dfrac{K}{S(S+2)}$

$\sigma = \dfrac{-2-0}{2-0} = -1$

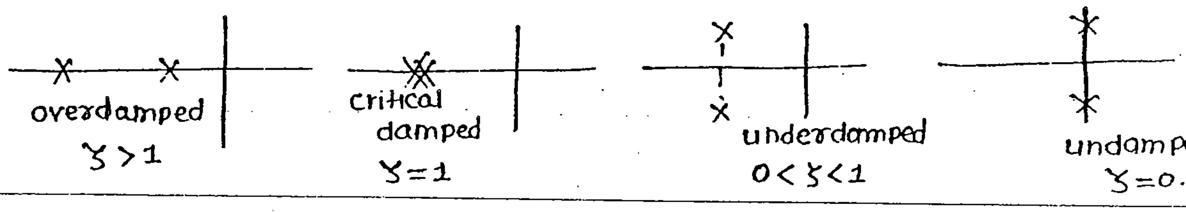
$\theta_1 = \dfrac{180^\circ}{2} = 90^\circ$

$\theta_2 = 270^\circ$



K=0
System become marginal stable

K>0
System become Stable



The above system is having ... the undamped. to get the
and underdamped nature but not the Systems, we required to
K-value for different nature of the Break point
apply magnitude condition at the

$$\left| \frac{K}{S(S+2)} \right|_{S=-1} = 1$$

$$\left| \frac{K}{(-1)(1)} \right| = 1$$

$$\underline{K = 1}$$

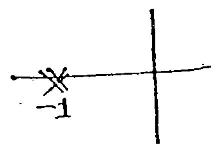
$0 < K < 1 \rightarrow$	overdamp.
$K = 1 \rightarrow$	Critical damp.
$K > 1 \rightarrow$	underdamp.

Put $\boxed{K=1}$

$\boxed{\text{Check}} \rightarrow$

$$G(S)\,H(S) = \frac{1}{S(S+2)}$$

$$CLTF = \frac{1}{S^2+2S+1} = \frac{1}{(S+1)^2}$$



* whenever RL diagram having more than or equal to 2, real axis RL Branches, the System Should have overdamped nature
* whenever RL diagram having Breakpoint then the system Should have Critical damped nature.
* whenever the RL diagram having Breakaway or Break in Point the System Should have under damped nature.
* whenever the angle of Asymptotes direction towards the imagenary axis and or the angle of arrival or departure direction towards the imagenary axis then System Should have undamped nature

②

$$G(S)\,H(S) = \frac{K}{S(S+1+j1)(S+1-j1)}$$

$P = 3.$

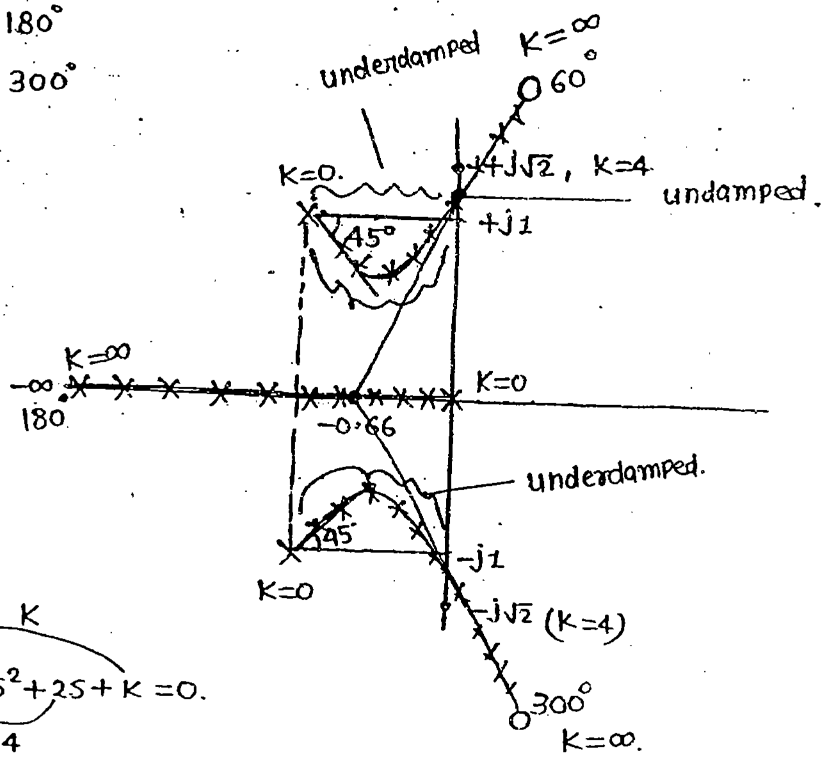
$Z = 0.$

$$\sigma = \frac{-2-0}{3} = -0.66$$

$$\theta_1 = \frac{180^\circ}{3} = 60^\circ$$

$$\theta_2 = 180^\circ$$

$$\theta_3 = 300^\circ$$



CE →

$$\overbrace{s^3 + \underbrace{2s^2 + 2s}_{4} + K}^{K} = 0.$$

$$\boxed{K = 4}$$

$$2s^2 + K = 0$$

$$\boxed{s = \pm j\sqrt{2}}$$

$$\angle GH \Big|_{s=-1+j1} = \frac{\angle K}{\angle s \;\; \angle(s+1+j1) \;\; \angle(s+1-j1)}$$

$$= \frac{0^\circ}{\angle 135^\circ \;\; \angle 0^\circ \;\; \angle 90^\circ} = -225^\circ$$

$$\phi_d = 180^\circ + \angle GH = \underline{\underline{-45^\circ}}$$

The above system is having only undamped and underdamped nature.

$\boxed{0 < K < 4}$ → underdamped

$\boxed{K = 4}$ → undamped.

$K > 4$ → unstable

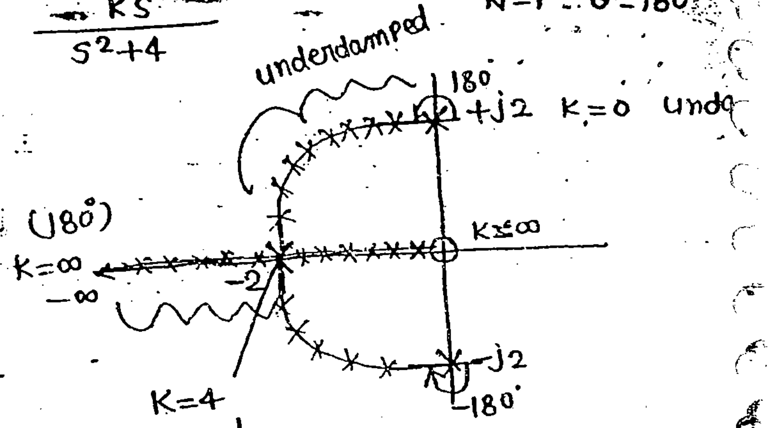
$$\angle G(S)\,H(S) = \frac{KS}{S^2+4}$$

N=1 ∴ θ=180

a

$$\sigma = \frac{0-0}{P-Z}$$

$$\boxed{\sigma = 0}$$



$$-1 = \frac{KS}{S^2+4}$$

$$K = \frac{-S^2-4}{S}$$

$$\frac{dK}{dS} = \frac{-2S(S)+S^2+4}{S^2} = 0$$

$$-S^2+4=0$$

$$\boxed{S=\pm 2}$$

$$\angle GH\Big|_{S=\pm j2} = \frac{\angle K\ \angle S}{\angle(S-j2)\ \angle(S+j2)} = \frac{\angle 0 \cdot \angle 90^\circ}{\angle 0\ \angle 90^\circ} = 0^\circ$$

$$\phi_d = 180^\circ + \angle GH = 180^\circ$$

K=0 undamped

0<K<4 − underdamped

K=4 critical damped

4<K<∞ overdamped.

$$\left|\frac{KS}{S^2+4}\right|_{S=-2} = 1$$

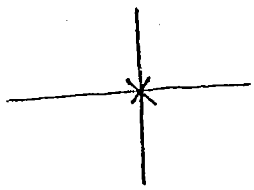
$$\rightarrow \boxed{K=4}$$

$\left\{\begin{array}{l} \text{B.P.},\ \sigma,\ \theta,\ \phi_d,\ \phi_a \\ \text{write Rules 9RL} \end{array}\right.$

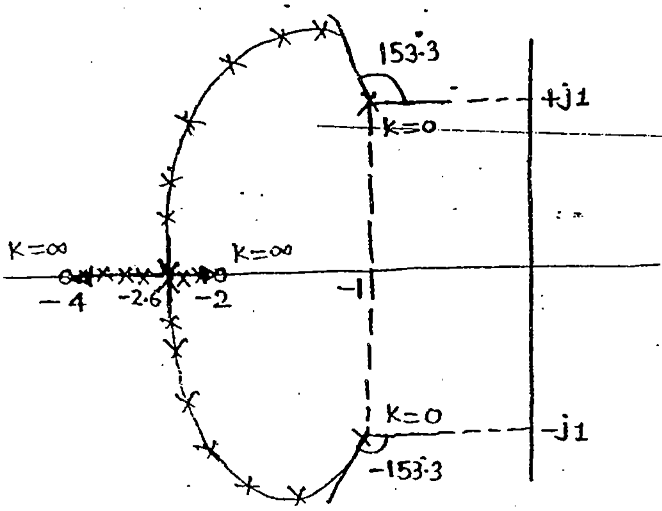
Q.

$$G(S)\,H(S) = \frac{K}{S}$$

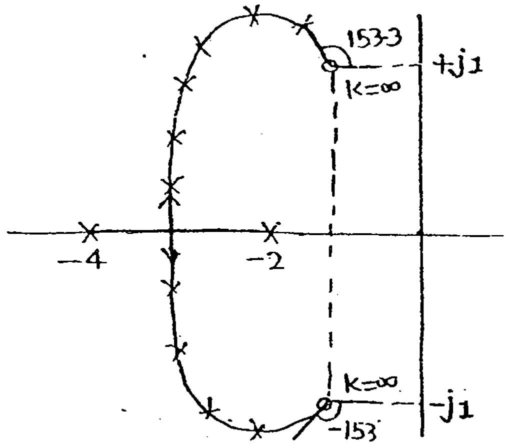
N=1 . θ = ±180°



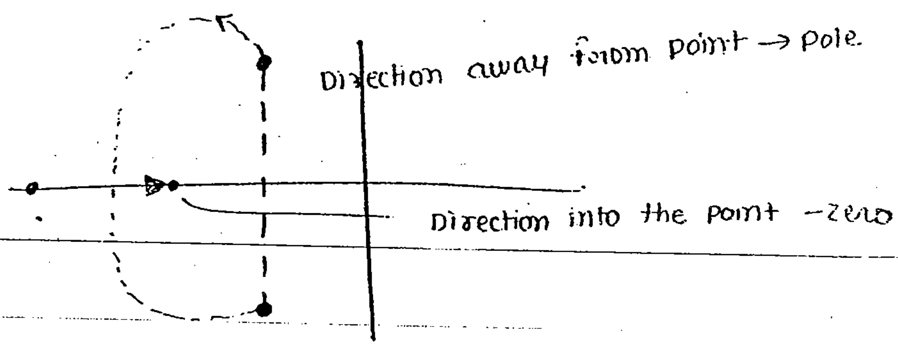
$$G(S)\,H(S) = \frac{K(S+2)(S+4)}{S^2+2S+2}$$



$$G(S)\,H(S) = \frac{K(S^2+2S+2)}{(S+2)(S+4)}$$



The correct RL Diagram is one which must be symmetrical about the Real axis and direction of RL Branches is from pole to zero.



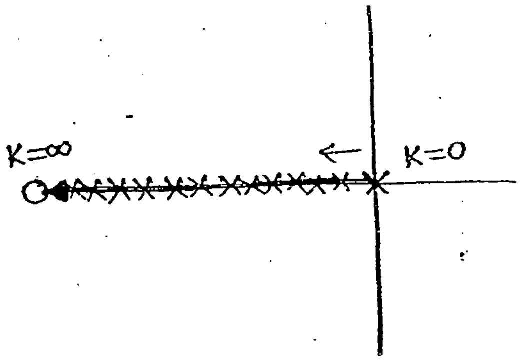
* whenever T.F. consist poles ... line is nothing but angle of asymptotes line

Q. $G(s)H(s) = K/s$

Angle of Asy:

$N = 1, \quad \theta = \frac{\pm 180^\circ}{N}$

$= \pm 180^\circ$



K=0 m.s

K>0 Stal

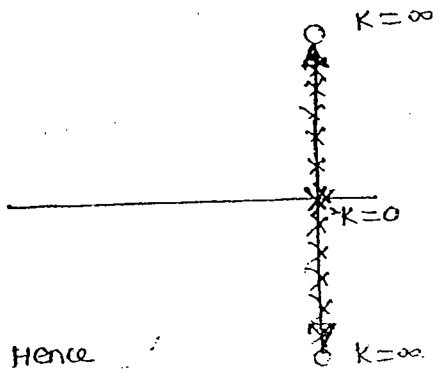
Q. $G(s)H(s) = K/s^2$

$N = P - Z = 2 - 0 = 2$

$\theta = \frac{\pm 180^\circ}{2}$

$\theta = 90^\circ$

$= 270^\circ$

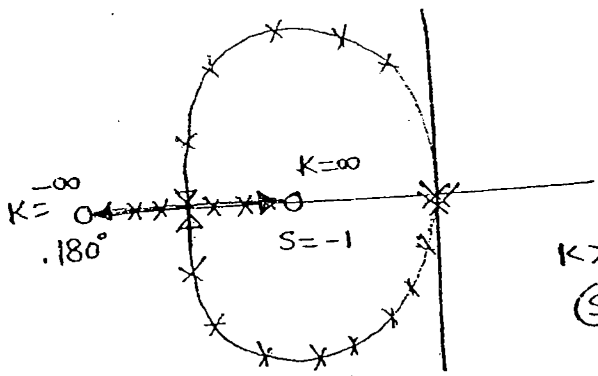


K>0 m.s.

The above sys. is m.s. Hence we require to make it stable by adding a one finite zero in the Left Hand Side

Q. $G(s)H(s) = \frac{K(s+1)}{s^2}$

$N = 1, \quad \theta = 180^\circ$



K>0

(S)

B.P.

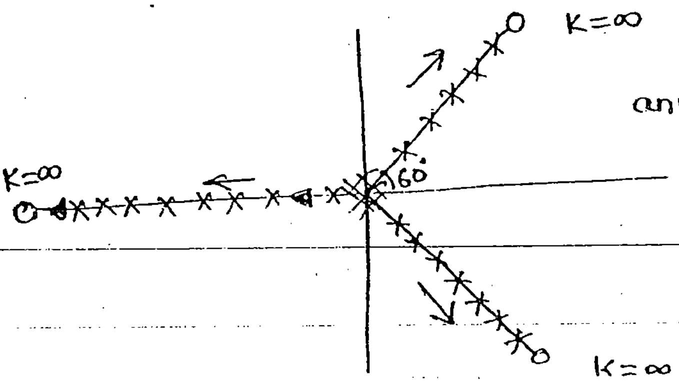
$-1 = \frac{K(s+1)}{s^2}$

$K = \frac{-s^2}{s+1}$

$\frac{dK}{ds} = 0$

$s = 0, -2$

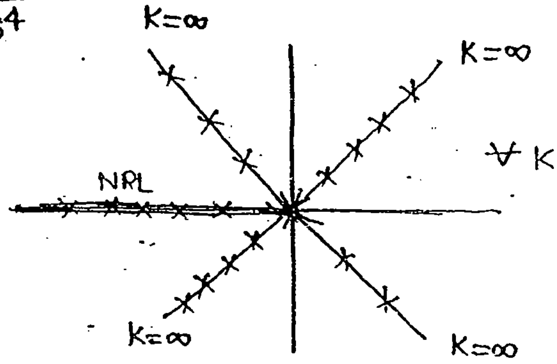
Q. $G(s)H(s) = \frac{K}{s^3}$



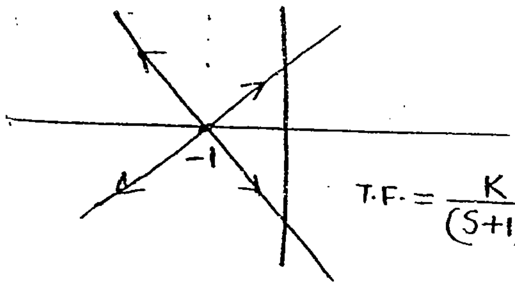
any V of sys ur

$\sigma = 0$

$$G(S)H(S) = \frac{K}{S^4}$$



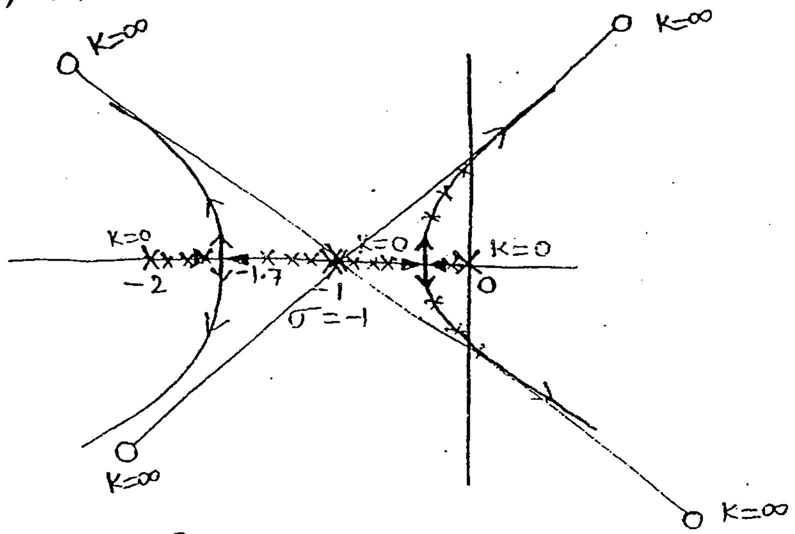
$\forall K \rightarrow$ US.



$$T.F. = \frac{K}{(S+1)^4}$$

Q. $$G(S)H(S) = \frac{K}{S(S+1)^2(S+2)}$$

B.P. = −0.29, −1, −1.7



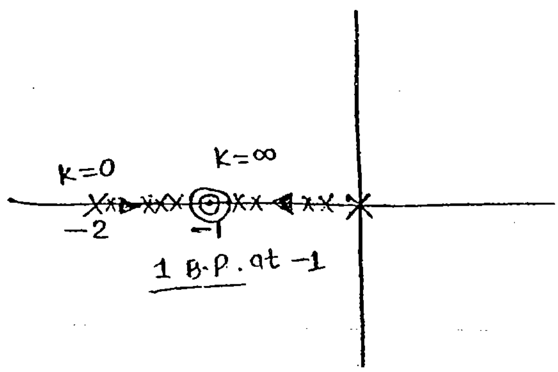
Q. $$G(S)H(S) = \frac{K(S+1)^2}{S(S+2)}$$

$P = 2$

$Z = 2$

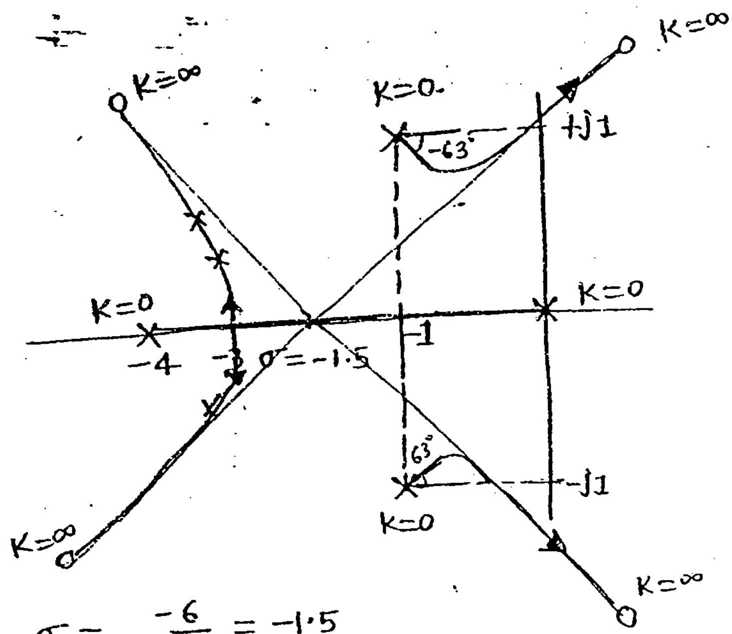
$N = 0$

so No need to calculate θ.



$$G(s)\,H(s) = \frac{K}{s(s^2+2s+2)(s+4)}$$

(Pole choose shortest path)



$\sigma = \frac{-6}{4} = -1.5$

B.P = −3.

$\theta = 45°, 135°, 225°, 315°$

$\phi_d = -63°.43$

$$\angle GH\Big|_{s=-1+j1} = \frac{\angle K}{\angle s \;\angle(s+1+j1)\;\angle(s+1-j1)\;\angle s+4}$$

$$= \frac{\angle 0°}{\angle 0 \;\angle 135° \;\angle 90° \;\angle 18.43°}$$

$= -243°.43$

$\phi_d = 180° - 243°.43 = -63°.4$

$$\boxed{\begin{array}{c} |B.P.| > |\sigma| \\ \phi_d < \mp 90° \end{array}}$$

GATE
***

The Root Locus Diagram Having complex conjugate poles and the magnitude of Break Point is greater than mag. of centroid then ϕ_d is less then $\mp 90°$ at complex pole

(near, paral enter, occupies the z(

Q. $$G(s)\,H(s) = \frac{K}{s(s+1)(s^2+2s+2)}$$

$$\sigma = \frac{\cancel{-1}-3-0}{4} = -0.75$$

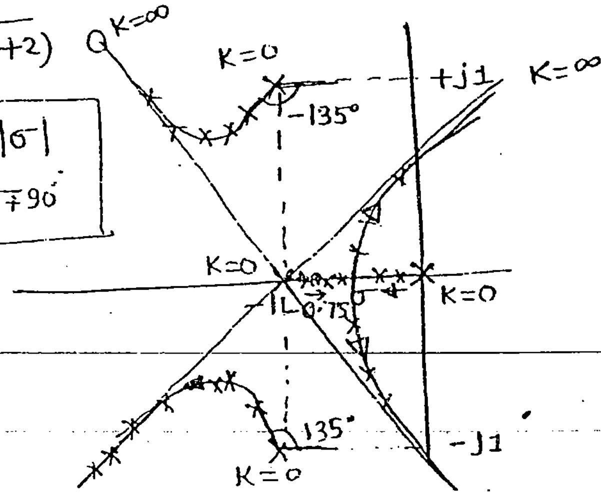
$$\boxed{\begin{array}{c} |BP| < |\sigma| \\ \phi_d > \mp 90° \end{array}}$$

$\theta = 45°, 135°, 225°$
$315°$

$\angle GH\Big| = \cancel{-63.43°+180°} = -315°$

B.P. = −0.39

$\phi_d = 180° - 315° = -135°$



$$G(S)H(S) = \frac{\quad}{S(S+2)(S^2+2S+2)}$$

$\sigma = -1$

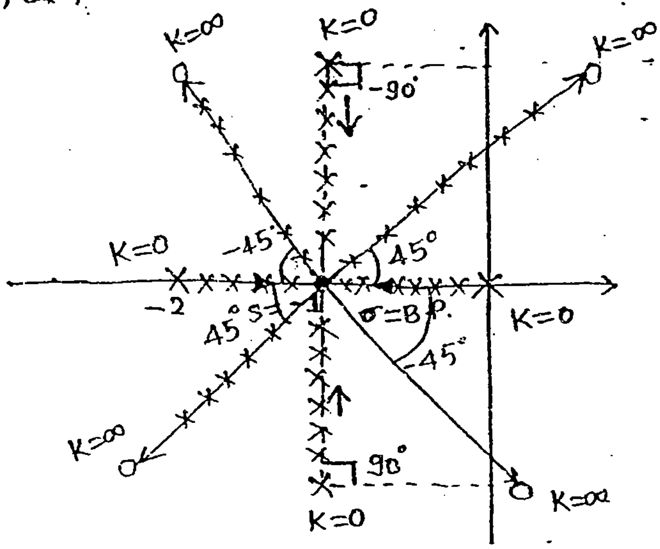
$\theta = 45^\circ, 135^\circ, 225, 315^\circ$

$B.P. = -1$

$$\boxed{\begin{array}{l}|BP| = |\sigma| \\ \phi_d = \mp 90^\circ\end{array}}$$

$B.P. = \sigma$

all the poles are symm. about B.P.



$$\theta = \frac{\pm 180^\circ}{4} = \pm 45^\circ$$

all poles moving oppo. direction with equal distance

→ in the above System no. of the poles meet at B.P. is 4.

→ the K Value at B.P. is $= 1$ (Apply mag. condition).

If B.P & centroid is equal & all poles are symmetric the poles will move to the centre

$$\left| \frac{K}{S(S^2+2S+2)(S+2)} \right|_{S=-1} = -1$$

$$\boxed{K = 1}$$

4 Pole at $S=-1$ so T.F.

→ The CLTF at B.P. $= \dfrac{1}{S(S+2)(S^2+2S+2)+1} = \dfrac{1}{(S+1)^4}$

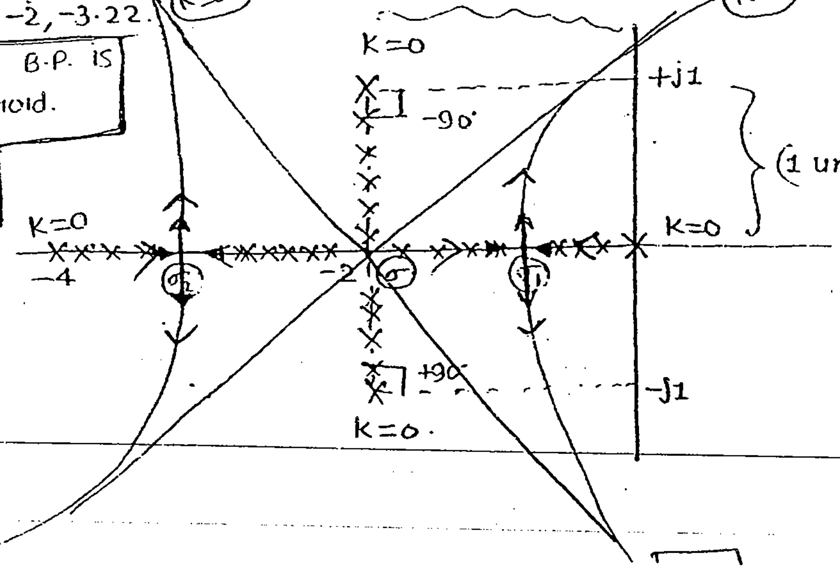
Q. $$G(S)H(S) = \frac{K}{S(S+4)(S^2+4S+5)}$$

$\sigma = -2$

B.P. $= -0.77, -2, -3.22$ (σ_1, σ, σ_2)

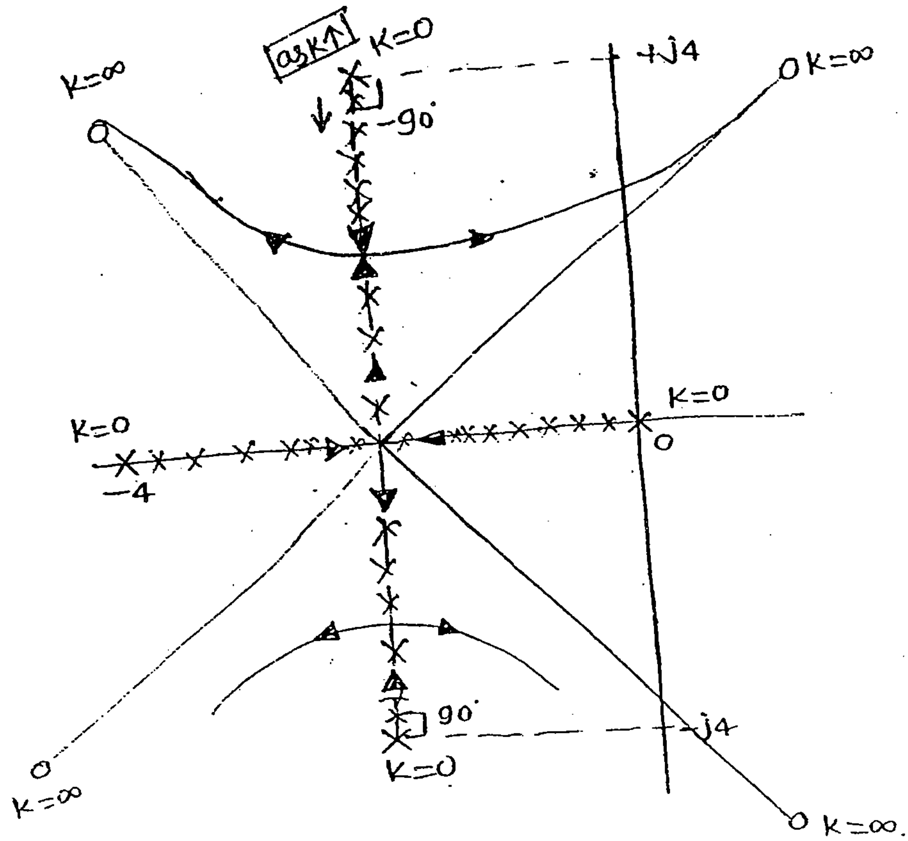
∵ one of the B.P. is equal to centroid.

B.P $= \sigma$ and complex Poles are very close to Real axis.



→ whenever complex poles are very closed to real axis then no. of Break points are increases.

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$



$\boxed{B.P. = \sigma}$

$\boxed{\phi_d = \mp 90°}$

B.P. = σ and complex poles far away from Real axis

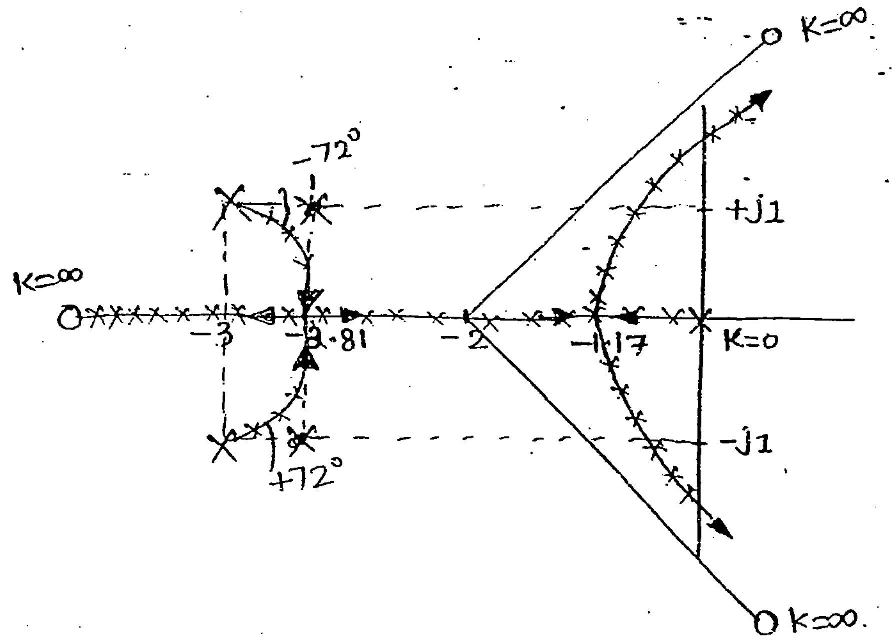
↓

1 Real B.A.P.
1 Pair of complex. B.A.P.

σ = Real part of Complex Pole

- Symmetrical → 1 B.P.
- Complex Pole very Close to Real axis → 3 R.A.B.A.P, 1 B.I.P. & 2 B.A.P.
- complex Pole are far away → 3 B.P.'s → 1 R.A.B.A. and 2 Complex Pole Break Away Poin

$$G(S)\,H(S) = \frac{K}{S\,(S^2+6S+10)}$$



$$G(H) = \frac{K(S+1)}{S^2(S+K_1)}$$

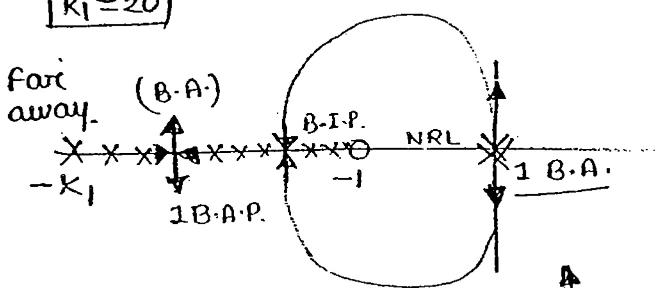
$K_1 = 20$ — 3 B.P.

$K_1 = 9$ — 2 B.P. } (S)

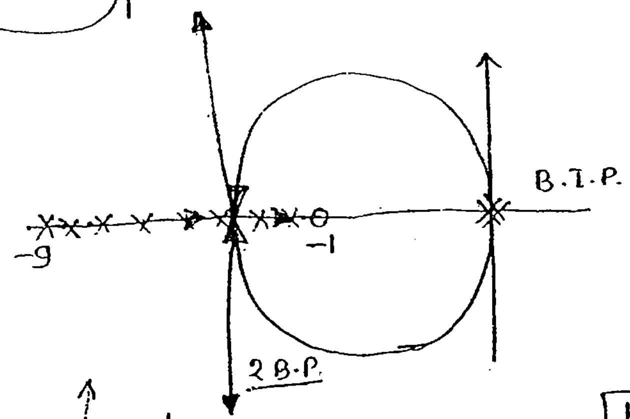
$K_1 = 2$ — 1 B.P.

$K_1 = 0.1$ — 1 B.P. } (US)

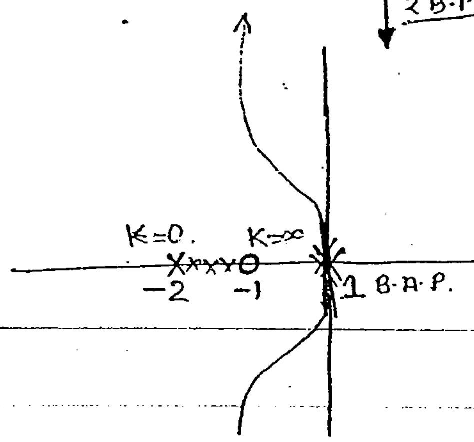
$\boxed{K_1 = 20}$



$\boxed{K_1 = 9}$

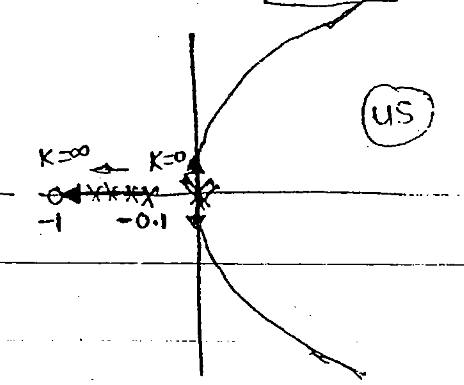


$\boxed{K_1 = 2}$



$\boxed{K_1 = 0.2}$

(US)



* Root Contours *

Whenever the T.F. are CE. (char. eqⁿ) consist the multiple unknown parameters. by varying all the parameters from -0 to ∞ and drawing a Root locus diagram is nothing but, Root contour.

→ Draw the Root contour to the given char. equation.

$$CE. = \underbrace{s^2 + as}_{deno.} + \overbrace{K}^{Num.} = 0. \quad \text{—— (1)}$$

for each value of a, K

Case-(1) 'K' as system gain (varies from K=0 to ∞).

a as constant. —— (a=0, 1, 2, 3 - - -

Case-(2) 'a' as system gain (varies from 0 to ∞).

'K' as constant.

NOTE — To draw a RL diagram, the system gain and its Related (parameters) terms are in the numerator, remain all the terms in the denominator.

Eq.(1) $[\div (s^2+a)]$

$$1 + \frac{K}{s^2+as} = 0 \qquad 1 + GH = 0.$$

for system (S)

a > 0

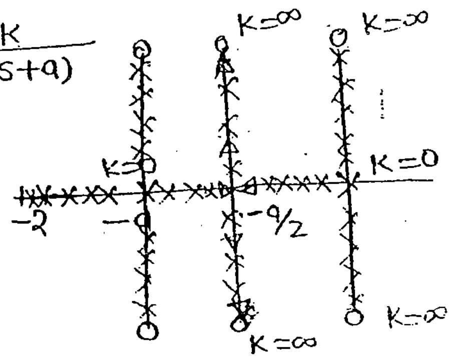
K > 0.

or

$$\frac{K}{s^2+as} = GH.$$

Case-I

$$GH = \frac{K}{s(s+a)}$$



family of curves.

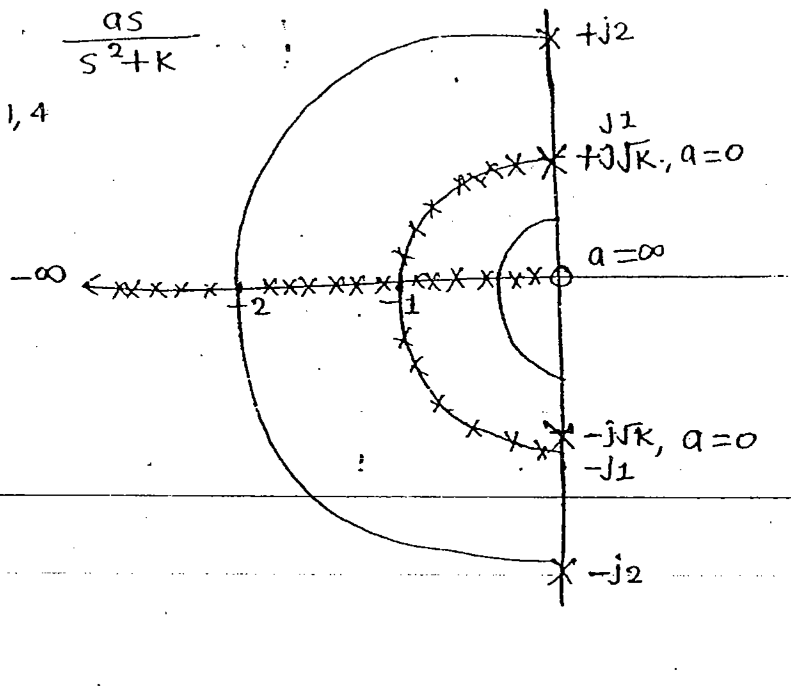
Case-II

$$GH = \frac{as}{s^2+K}$$

K = 1, 4

a > 0

K > 0



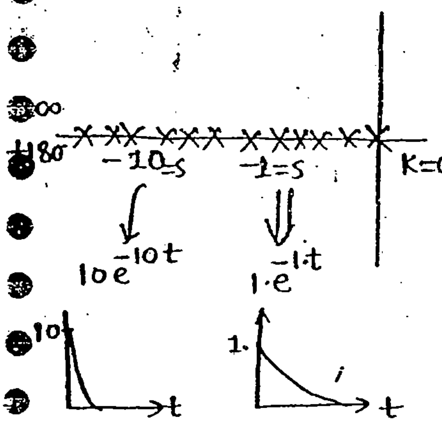
* Effect of addition of Poles and zero-

The addition of Poles and zero's only L-H of S-Plane

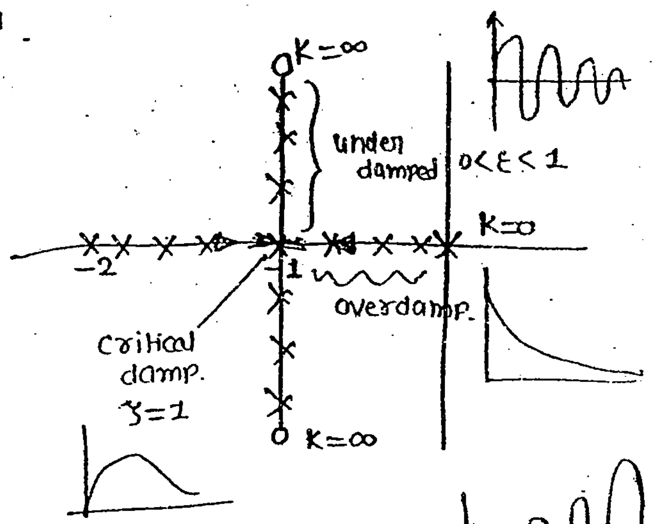
Addition of Poles-

$GH = K/S$ $\quad GH = 1/S$ $\quad G(S)H(S) = \frac{K}{S(S+2)}$

$N=1$ $\quad \theta = \pm 180°$ $\quad CL = \frac{1}{S+1}$

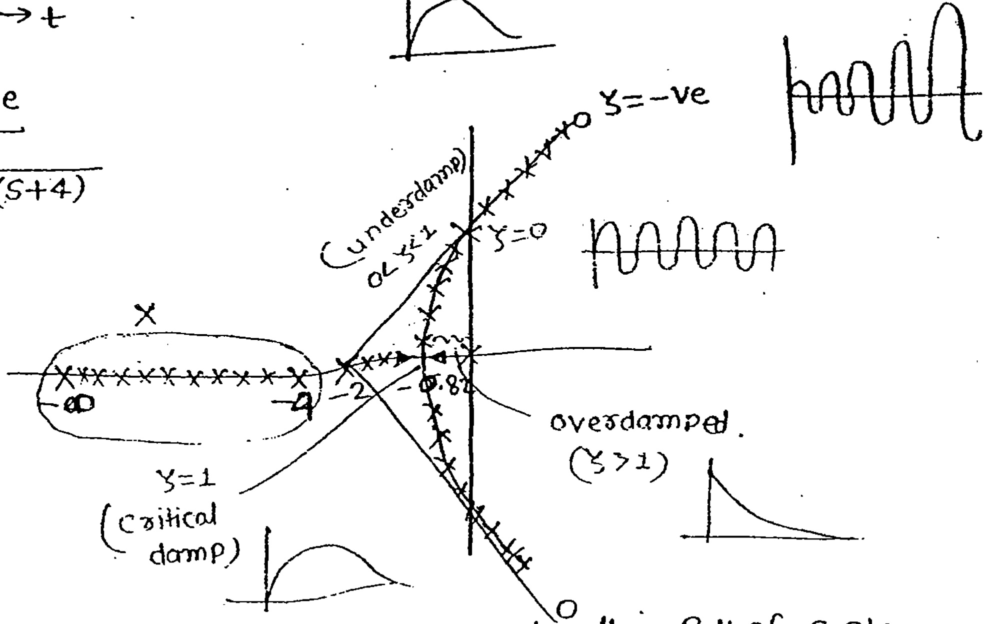


K>0 (S)
exp. decay
No osc.



Add more pole

$$GH = \frac{K}{S(S+2)(S+4)}$$



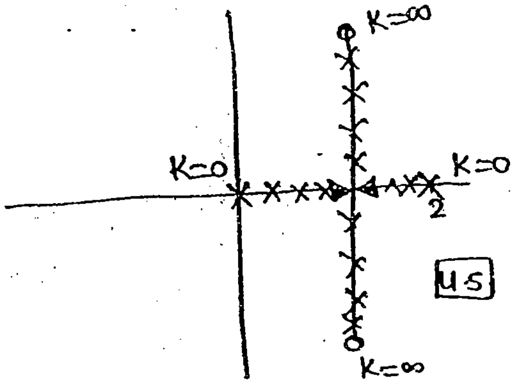
1) The Root Locus Branches shifted towards the R-H of S Plane
2) The B.P. shifted towards Imagenary axis.
3) The Relative stability ↓ ④ the system become more oscillatory
4) The range of K value for system stability decreases.
5) ξ (damping factor) ↓
as $\xi\downarrow \rightarrow \omega_d \uparrow \rightarrow$ $\boxed{t_r, t_d, t_p \downarrow}$ means Transient performance is improved.
6) % m_p ↑↑ so that system become less stable
7) Time constant ↑↑ so settling time also increases
8) ... because $t_r\downarrow$. The Response Becomes Very quick.

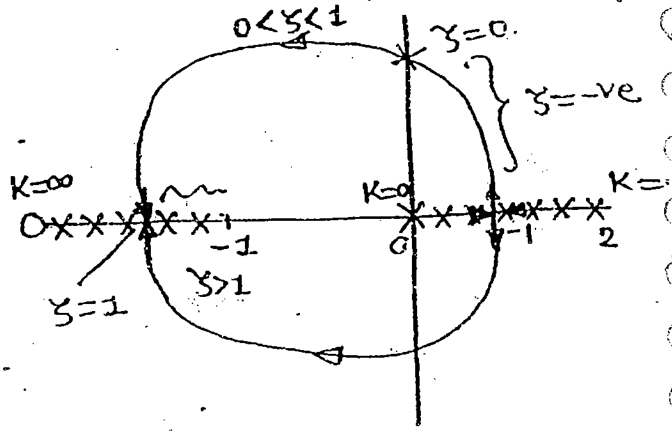
Addition of zero

$$GH = \frac{K}{S(S-2)}$$

$$GH = \frac{K(S+1)}{S(S-2)}$$

N = 1
θ = 180°





① Root Locus Branches shifted towards Left Half of S-plane.

② B.P. Shifted towards the added zero.

③ R.S. Improved.

④ System become less oscillatory.

⑤ Range of K increases, System Stability increases.

⑥ $\zeta \uparrow$ then $\omega_d \downarrow \rightarrow \boxed{t_r, t_d, t_p \uparrow}$ Transient performance decreas

⑦ % $M_p \downarrow\downarrow$ so that system become more stable

⑧ Time constant $\downarrow\downarrow$ so settling time also decrease.

⑨ BW $\downarrow$ because $t_r \uparrow$, the response become very slow.

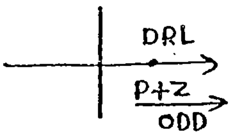
The addition of zero in left most side Improves the Steady State performance. (addition of zero = zero added In the left most side).

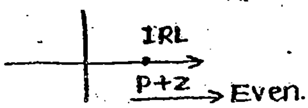
Note -- If zero is added near to the imagenary axis, the transient performance is improved similar to the effect of addition of poles. (Inverse Root Locus)

※ Draw the Root Locus diagram.

$$GH = \frac{K e^{-S}}{S(S+1)}$$

	DRL	IRL
R_1	Symmetrical	〃—〃
R_2	No. of loci	〃—〃
R_3	Real axis loci	





a point to exist on IRL Branch the sum of poles and zero to the Right Hand side of that point should be even.

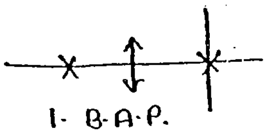
R_4. Asymptotes.

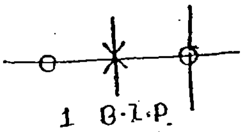
DRL: $\theta = \dfrac{(2q+1)\,180^\circ}{P-Z}$

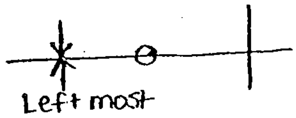
IRL: $\theta = \dfrac{2q \times 180^\circ}{P-Z}$

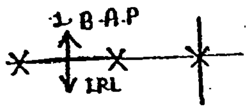
R_5 centroid — 〃—〃

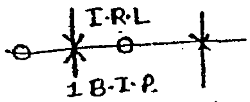
R_6 Breakpoint.

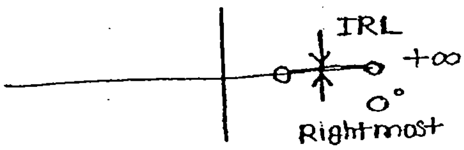












whenever there exist the Rightmost side zero to the rightmost side of zero there exist a Inverse Root locus Branch then there should be minimum one Breakin point to the Right most side of zero

R_7 intersection point with Imagenary axis — 〃—〃

R_8 Angle of departure –

DRL:

$\phi_d = 180^\circ + \angle GH$

$\phi_a = 180^\circ - \angle GH$

IRL:

$\phi_d = 0^\circ + \angle GH$

$\phi_a = 0^\circ - \angle GH$

Q. $G(s)H(s) = \dfrac{Ke^{-s}}{s(s+1)} = \dfrac{K(1-s)}{s(s+1)} = \dfrac{-K(s-1)}{s(s+1)}$

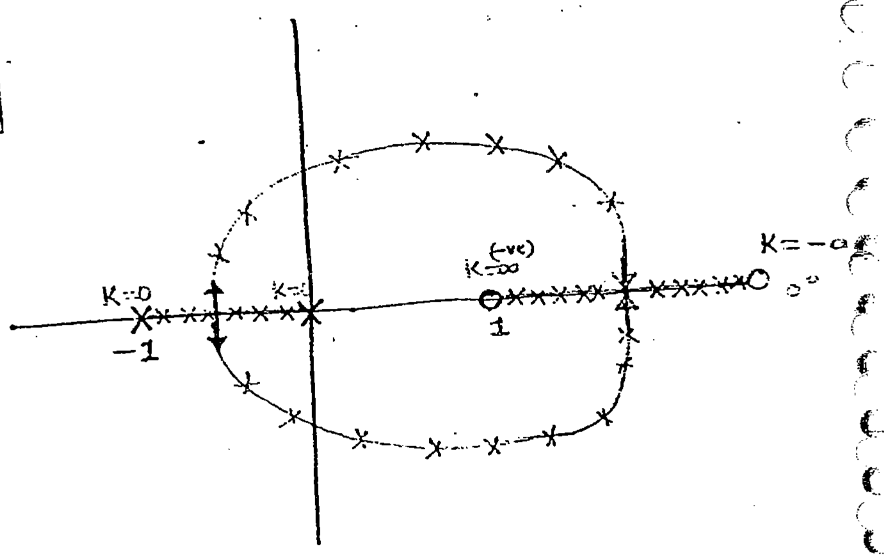
Any T.F. having exp. term then it is called Transportation delay or lag system. The Transportation delay system effect the phase but not the magnitude

-ve f/b.

$1 + GH = 0$

$1 - \dfrac{K(s-1)}{s(s+1)} = 0.$

(IRL)



*

to draw a RL Diagram in the T.F. the s-term should not have the -ve sign

GH = +K( )	
-ve (f/b)	+ve (f/b)
$1+GH=0$	$1-GH=0$
$1+K(\,)=0$ ↑ DRL	$1-K(\,)=0$ ↑ IRL

G(s)H = -K( )	
-ve f/b (Def)	+ve f/b
$1+GH=0$	$1-GH=0$
$1-K(\,)=0$ ↑ IRL	$1+K(\,)=0$ ↑ DRL

$N = 1$, $\theta = \dfrac{(2q)180^\circ}{(P-Z)} = 0^\circ$

$(-\infty \text{ to } 0) \rightarrow$ IRL

$(0 \text{ to } +\infty) \rightarrow$ DRL

Draw the Complete Root Locus. (Range of K is $-\infty$ to $+\infty$)

$G(s)H(s) = \dfrac{K}{s(s+2)}$

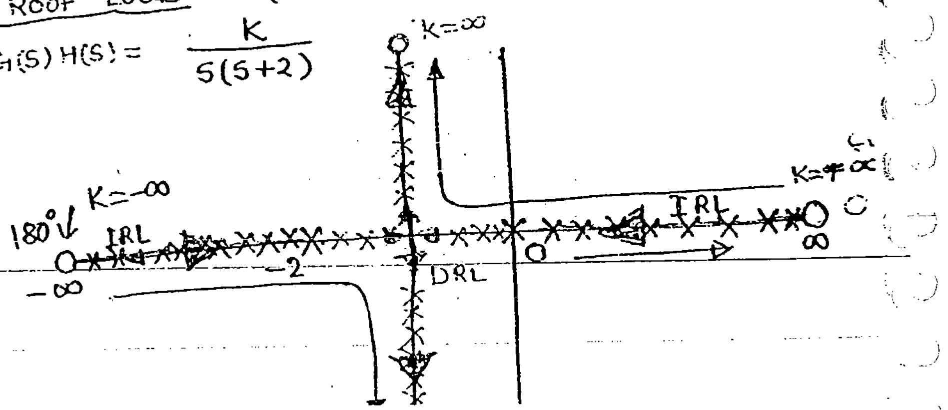
IRL $\quad 0,1$

$\theta = \dfrac{(2q)180^\circ}{P-Z} = 0^\circ, 180^\circ$

DRL $\quad 0,1$

$\theta = \dfrac{(2q+1)180^\circ}{P-Z}$

$= 90^\circ, 270^\circ$



→ to show the continuous Root locus Branch from (−∞ to
we require to change the direction in the ~~...~~

BODE PLOT

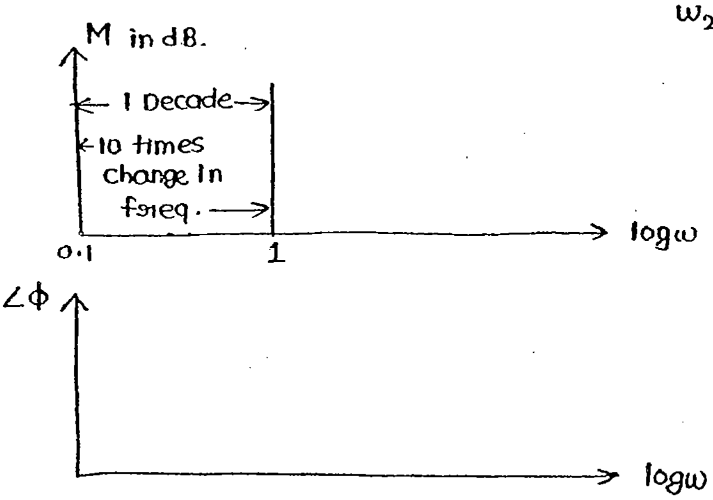
Purpose -

* To Draw the frequency response of an O.L.T.F.
* To Find the closed loop system stability.
* To Find the gain margin, phase margin, gain crossover freq. and phase crossover frequency (ω_{pc})
* To Find the Relative stability, the largest G.M. and P.M. gives the more Relatively stable, but system response become slow, the $\boxed{\text{smallest G.M. and P.M.}}$ gives the less relatively stable, but system response become ~~fast~~ oscillatory.

optimum value of G.M. = 5 to 10 dB.
" ——— " P.M. = 30° to 40°

* The Bode plot consist the two plot
  (a) magnitude plot
  (b) phase plot

$\omega_2 = 10\omega_1$ (Decade
$\omega_2 = 2\omega_1$ (octave



* Procedure to draw the Bode plot —

(1) s Replaced by $j\omega$, to convert into freq. domain
(2) Find the magnitude and write in dB.

$$M_{in\ dB} = 20 \log |G(j\omega)H(j\omega)|$$

(3) Find the phase Angle by

$$\phi = \tan^{-1}\left(\frac{\text{Img. Part}}{\text{Real part}}\right)$$

(4) Vary freq. from 0 to ∞, draw the mag. and phase plot.

Step -(I)

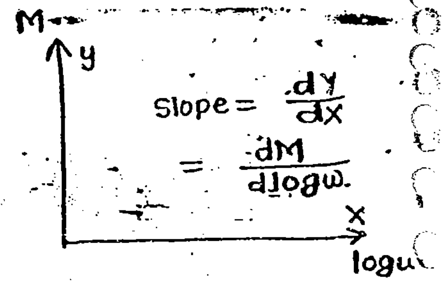
$$G(S)\,H(S) = K$$

$$G(j\omega)\,H(j\omega) = K$$

$$M = K$$

$$M_{dB} = 20\log K$$

$$\angle G(S)H(S) = \angle K = 0^\circ$$

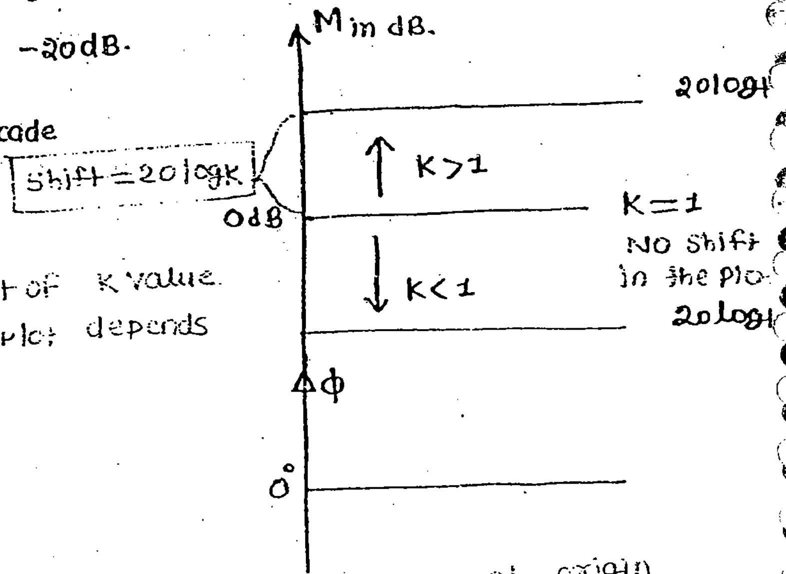


$$\text{Slope} = \frac{dy}{dx} = \frac{dM}{d\log\omega}$$

$K = 1$	$M_{in\ dB} = 0\ dB.$
$K > 1$	$M_{in\ dB} = +20\,dB.$
$K < 1$	$M_{in\ dB} = -20\,dB.$

$$S = \frac{dM}{d\log\omega} = 0\ dB/decade$$

$$\angle G(j\omega)H(j\omega) = \angle K = 0^\circ$$



* phase plot is independent of K value.
* whereas shift in mag. plot depends on K value

N- Poles at origin

$$G(S)H(S) = \frac{1}{S^n}$$

$S \to j\omega$

$$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)^n}$$

$$M = \frac{1}{\omega^n}$$

$$(M)_{dB} = -20n\log\omega.$$

$$\text{slope} = \frac{dM}{d\log\omega} = -20n$$

$$\angle\phi = \frac{\angle 1}{\angle j\omega \ldots n \text{ times}} = \frac{0}{90^\circ \ldots n\text{ times}}$$

$$= -90^\circ n.$$

N- Zero at origin.

$$G(S)\,H(S) = S^n.$$

$S \to j\omega$

$$G(j\omega)\,H(j\omega) = (j\omega)^n$$

$$M = (\omega)^n$$

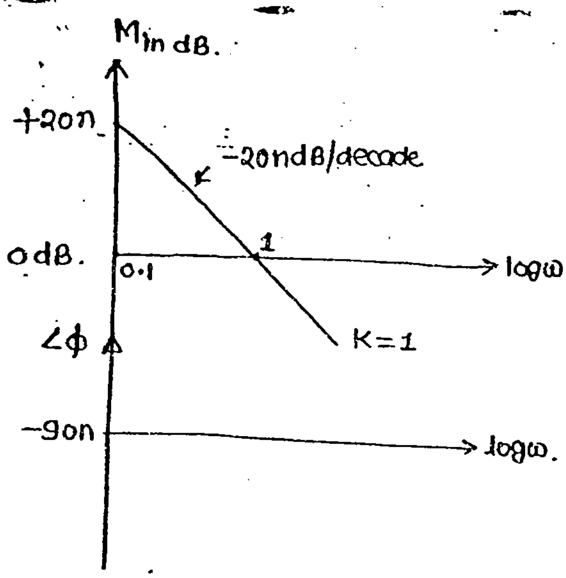
$$(M)_{dB} = 20n\log\omega.$$

$$S = \frac{dM}{d\log\omega} = +20n\ dB/dec$$

$$\angle\phi = \angle j\omega \ldots n \text{ times} = +$$

Note-

whenever the T.F. consist the Poles and zero at origin then magnitude plot starts with mag. of oppo. sign of slope. at a freq. of 0.1 and It should be passes ~~through~~ 0 dB line intersect at $\omega = 1$ extended upto ∞, If no corner freq. exist.



n- finite poles -

$$G(s)H(s) = \frac{1}{(s\tau+1)^n}$$

- s = -20n dB/dec.
- ∠φ = -90n.

$$G(j\omega)H(j\omega) = \frac{1}{(j\omega\tau+1)^n}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\left(\sqrt{\omega^2\tau^2+1}\right)^n}$$

$$\boxed{|G(j\omega)H(j\omega)|_{dB} = -20n \log\sqrt{(\omega\tau)^2+1}}$$

$$\phi_{actual} = \frac{\angle 1}{\angle(j\omega\tau+1) \cdots n \text{ times}}$$

$$\boxed{\phi_{actual} = -n \tan^{-1}(\omega\tau)}$$

Asymptotic/ Approx:-

Case-I $\omega\tau < 1$ (Neg. $\omega\tau$)

$M_{asy.} = 0$ dB , $s = 0$ dB/dec.

$$\phi_{asy} = \frac{\angle 1}{\angle 1} = 0^\circ$$

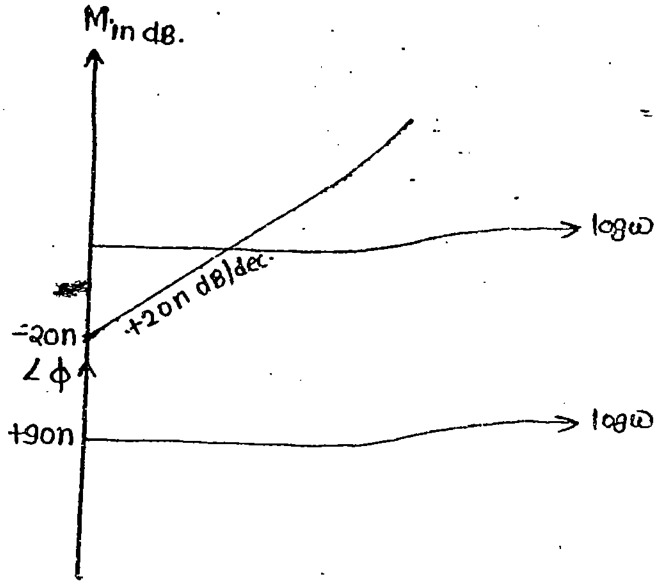
Case-II $\omega\tau > 1$ (Neg. 1)

$$M_{asy} = -20n \log(\omega\tau)$$

$$= -20n \log(\omega) - 20n \log\tau$$

$$s = \frac{dM}{d\log\omega} = -20n \text{ dB/dec.}$$

$$\phi_{asy} = \frac{\angle 1}{\angle j\omega\tau} = -90^\circ n$$



n- finite zero's

$$G(s)H(s) = (s\tau+1)^n$$

$$G(j\omega)H(j\omega) = (j\omega\tau+1)^n$$

$$\boxed{|G(j\omega)H(j\omega)|_{dB} = 20n \log\sqrt{(\omega\tau)^2+1}}$$

$$\phi_{actual} = \angle(j\omega\tau+1) \cdots n \text{ times.}$$

$$\boxed{\phi_{actual} = n \tan^{-1}(\omega\tau)}$$

→ Asymptotic/ Approximation -

Case-I

$\omega\tau < 1$ (Neg. $\omega\tau$)

$M_{asy.} = 0$dB, $s = 0$ dB/dec.

$$\phi_{asy} = \frac{\angle 1}{\angle 1} = 0^\circ$$

Case-II -

$\omega\tau > 1$ (Neg. 1)

$$M_{asy.} = 20n \log(\omega\tau)$$

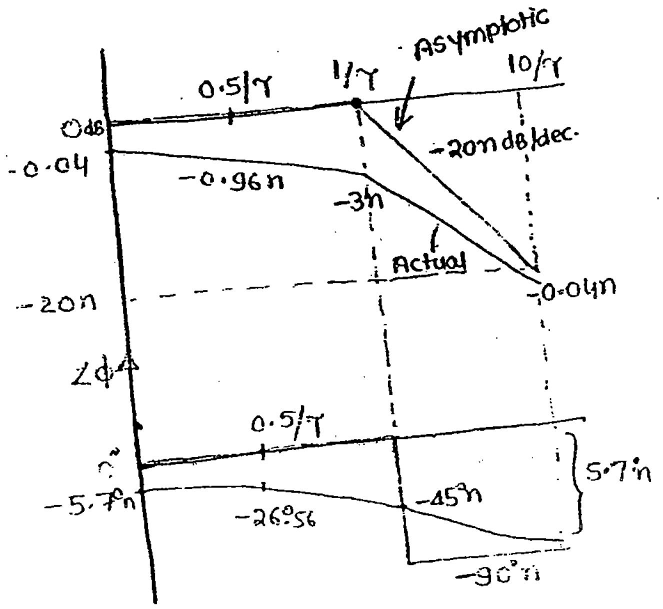
$$= 20n \log\omega + 20n \log\tau$$

$$s = \frac{dM}{d\log\omega} = 20n \text{ dB/dec.}$$

$$\phi_{asy} = \frac{\angle j\omega\tau}{1} = 90n$$

The frequency at which slopes
..nges from one level to
..nother level.
corner freq. is nothing But
..inite poles and finite zero location
in form of magnitude

	Slope	ϕ
$<$ CF	0 dB/dec.	0°
$>$ CF	$-20n$ dB/dec.	$-90^\circ n$.



Error = Actual value − Asymptotic value

Error in the magnitude plot −

Error at CF $\Big|_{\omega=1/T}$ =

$$\{M_{actual}\}_{\omega=1/T} - \{M_{asymptotic}\}_{\omega=1/T}$$

M_{actual} value → obtained from the T.F.
Asymptotic value → taken from plot

$\Rightarrow -3n - 0$

$\boxed{E = -3n}$ dB.

$E_{at\ \omega=0.5/T} = -0.96n - 0 = -0.96n$

Error in phase plot

$$\left(\text{Error at CF}\right)\Big|_{\omega=1/T} = \phi_{actual}\Big|_{\omega=1/T} - \phi_{asy}\Big|_{\omega=1/T}$$

$$= -45^\circ n - 0 = -45^\circ n$$

Error at $\omega = \frac{0.5}{T}$ (take the lower value of 0° and 90°)

→ at the corner freq. Error is max.

* either above or below CF, error decreases Symmetrically

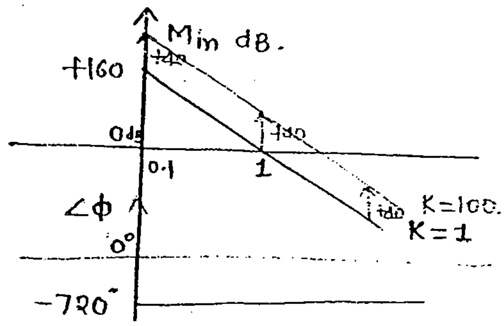
* Draw the Bode Plot for

$$G(S)H(S) = \frac{100}{S^8}$$

$$G(j\omega)H(j\omega) = \frac{100}{(j\omega)^8}$$

$$8P \Rightarrow -20\cancel{n}^{\,8} = -160\ dB/dec.$$

$$-90\cancel{n}^{\,8} = -720^\circ$$



Shift = $20 \log 100$

$= 40\,dB.$

* $$G(S)H(S) = \frac{100(S+5)^2}{S(S+2)(S+10)}$$

$$= \frac{\overset{5}{\cancel{100}} \times 25}{\cancel{20}} \left(\frac{(1+0.2S)^2}{S(0.5S+1)(0.1S+1)} \right)$$

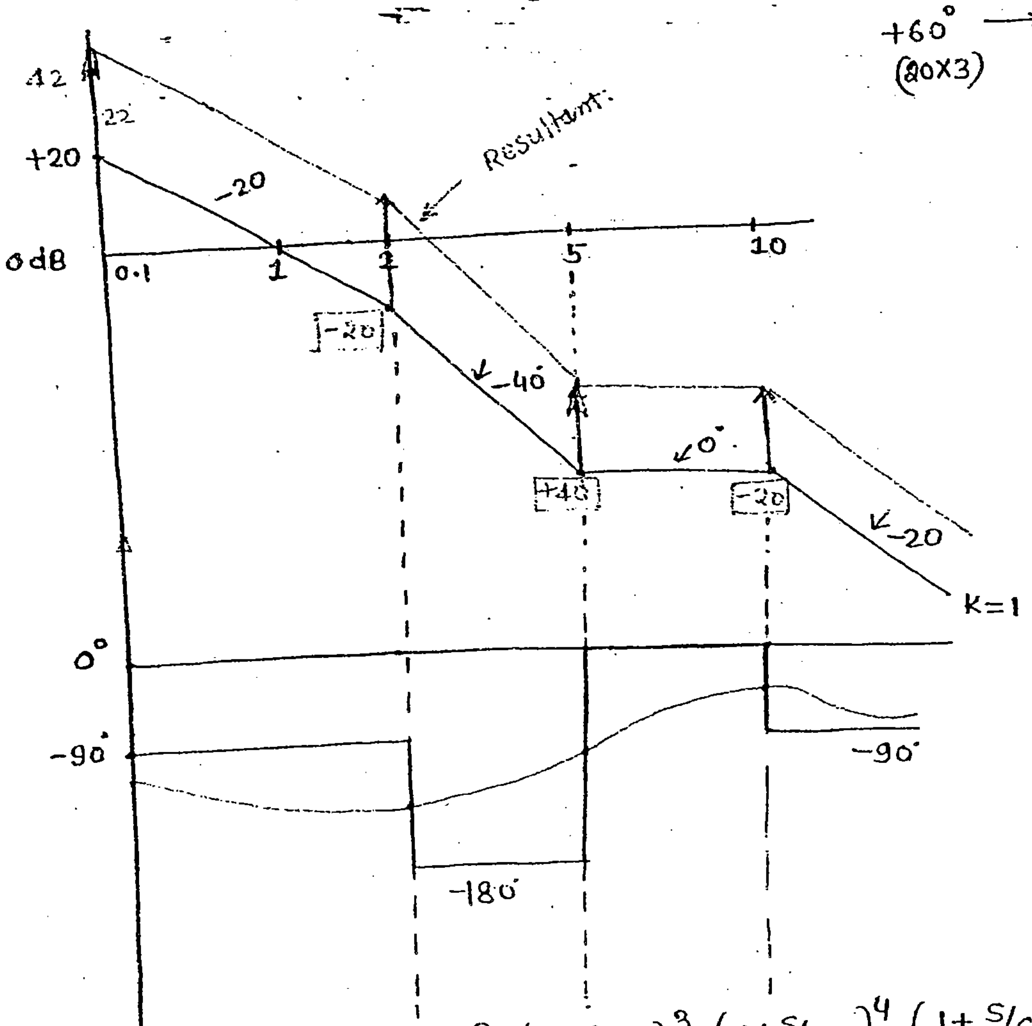
$$= \frac{12.5\left(1+\frac{S}{5}\right)^2}{S\left(1+\frac{S}{2}\right)\left(1+\frac{S}{10}\right)} \quad CF$$

The initial slope of the plot given by poles and zeros located at origin.

$-20n \rightarrow -90n$

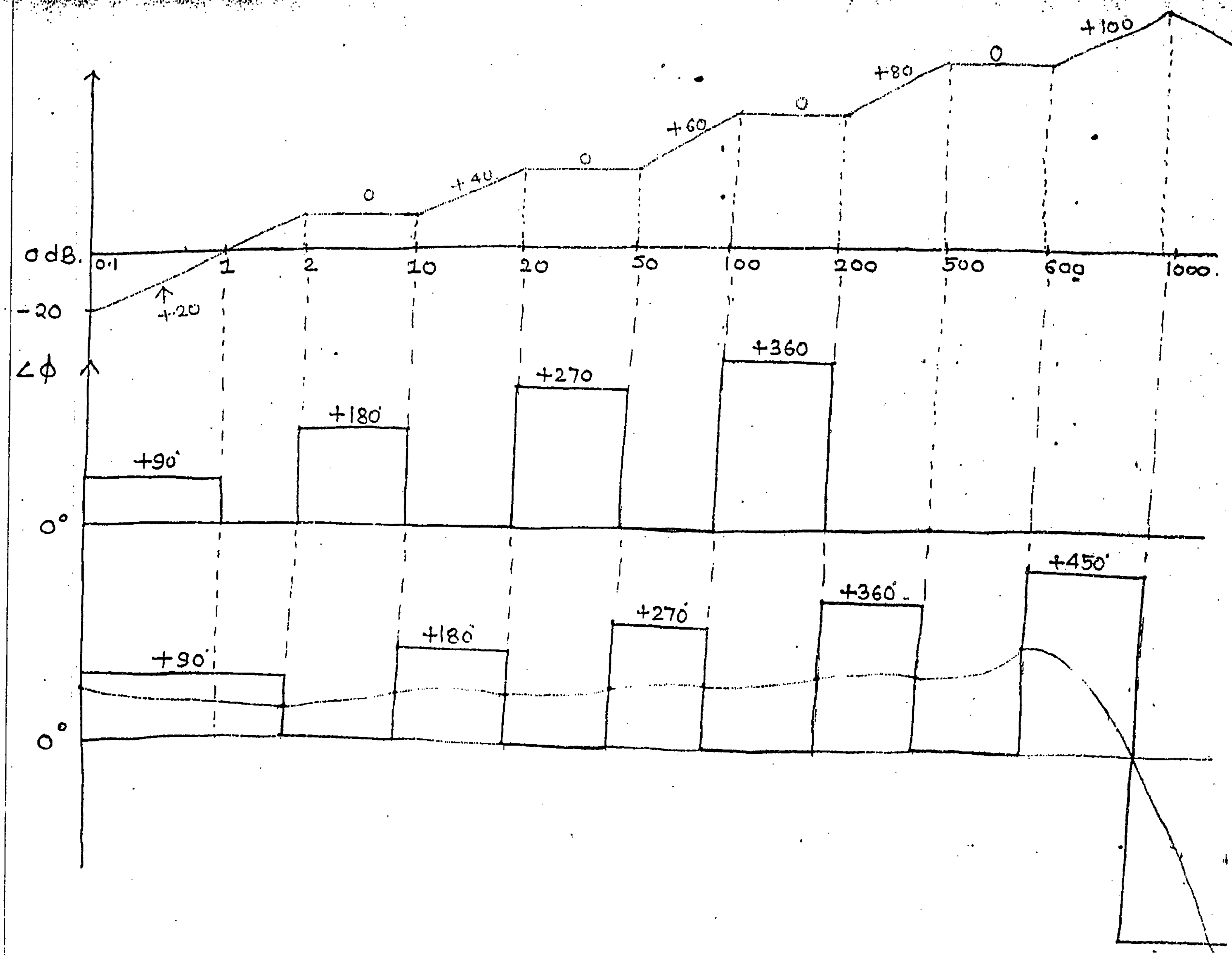
$+60^\circ \rightarrow +270^\circ$

(20×3)



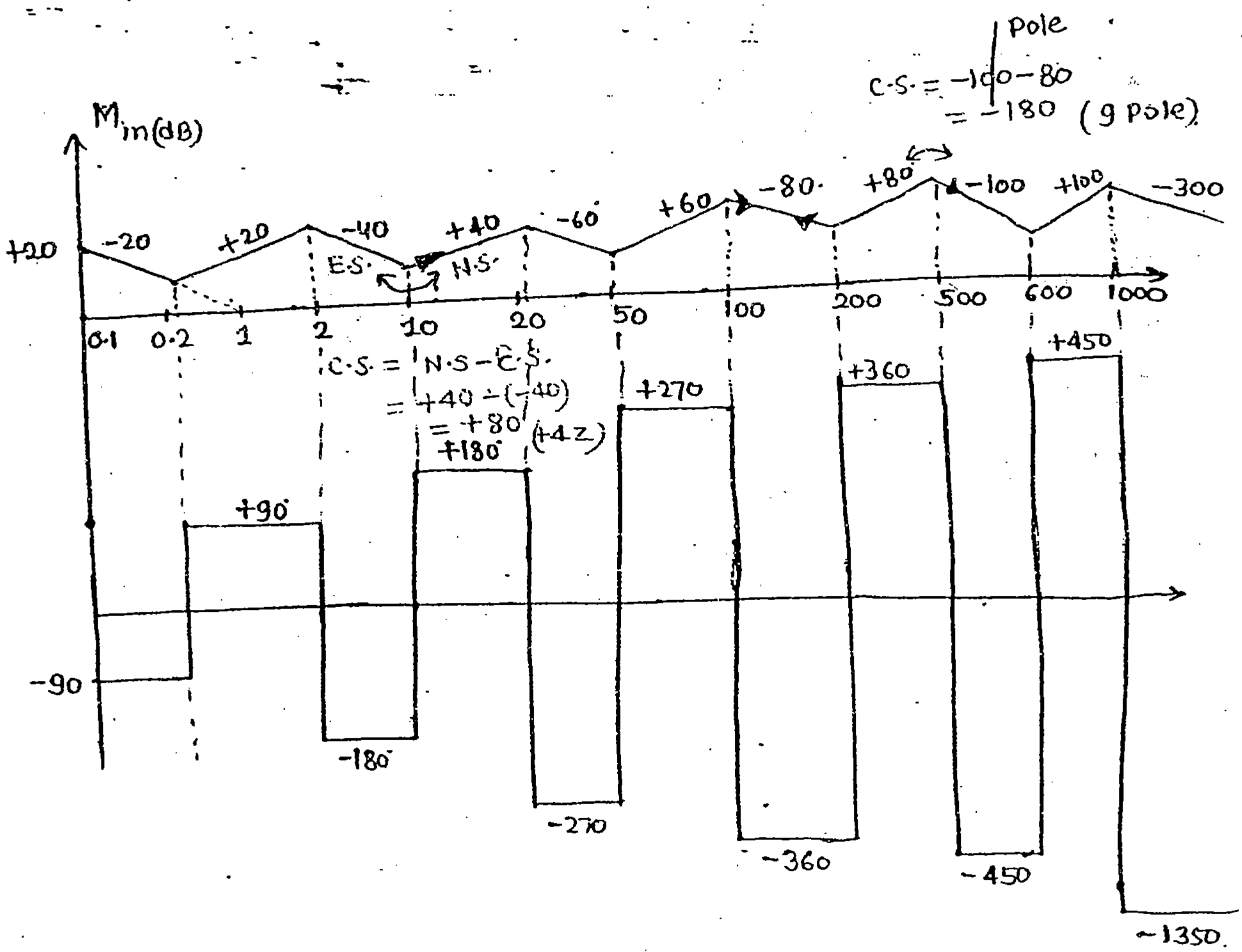
$$G(S)H(S) = \frac{s\left(1+S/10\right)^2 \left(1+S/50\right)^3 \left(1+S/200\right)^4 \left(1+S/600\right)^5}{\left(1+S/2\right)\left(1+S/20\right)^2 \left(1+S/100\right)^3 \left(1+S/500\right)^4 \left(1+S/1000\right)^1}$$

$$G(S)H(S) = \frac{\left(1+S/0.2\right)^2 \left(1+S/10\right)^4 \left(1+S/50\right)^6 \left(1+S/200\right)^8 \left(1+S/600\right)}{s\left(1+S/2\right)^3 \left(1+S/20\right)^5 \left(1+S/100\right)^7 \left(1+S/500\right)^9 \left(1+S/1000\right)}$$



+100
-100
0
+80
+60
+40
0 dB.
-20
+20
0.1
1
2
10
20
50
100
200
500
600
1000.
∠φ
+90°
+180°
+270
+360
0°
+450°
+360°
+270°
+180°
+90°
0°

0.2, 2, 10, 20, 50, 100, 200, 500, 600, 1000.



⊛ Find the change in slope at the following corner freq.

(i) $\omega = 2$ (ii) $\omega = 10$ (iii) $\omega = 20, 50, 100, 200, 500, 600, 1000.$

⊛ Find the slope of the line b/w two frequencies

$\omega = 2$ to 10, 20 to 50, 200 to 500, High frequency Asymp.

⊛ Find the slopes around the CF

(1) 2 (2) 20 (3) 200 (4) 1000

$$G(S)H(S) = \frac{s^3\left(1+S/10\right)^{12}\left(1+S/50\right)^{20}\left(1+S/200\right)^{40}\left(1+S/600\right)^{100}}{\left(1+S/2\right)^{5}\left(1+S/20\right)^{15}\left(1+S/100\right)^{25}\left(1+S/500\right)^{50}\left(1+S/1000\right)^{2}}$$

change in slope at any corner freq = slope of no of finite zero at the corner freq. pole, slope of no. of finite

ω	P/Z	C.S.
ω = 2	5P	-100
2	12Z	+240
10	15P	-300
20		+400

100	25P	−500
200	40Z	+800
500	50P	−1000
600	100Z	2000
1000	200P	−4000

10 to 20

>10 & <20
(Included) (excluded)

4P
6Z
2Z → +40 dB/dec.

>100 & <200
(Included) (excluded)

16P
12Z
4P → −80 dB/dec.

>2 & <10
3Z
5P
2P → −40.

>20 & <50
15Z
20P
5P → −100

>200 to <500
75Z
45P
30Z → +600

High frequency asymptotes = ∞.

175 Z
295 P
120 P −2400 dB/dec.

(iii) around (2)

<2
(excluded)

3Z → +60

>2
(Included)

3Z
5P
2P → −40 dB/dec.

around 20

<20
(excluded)

5P
15Z
10Z
200 dB/dec.

>20
(Included)

20P
15Z
5P −100 dB/dec.

(iv) around 200

<200
(excluded)

~~38Z~~
49P
10P
−240 dB/dec.

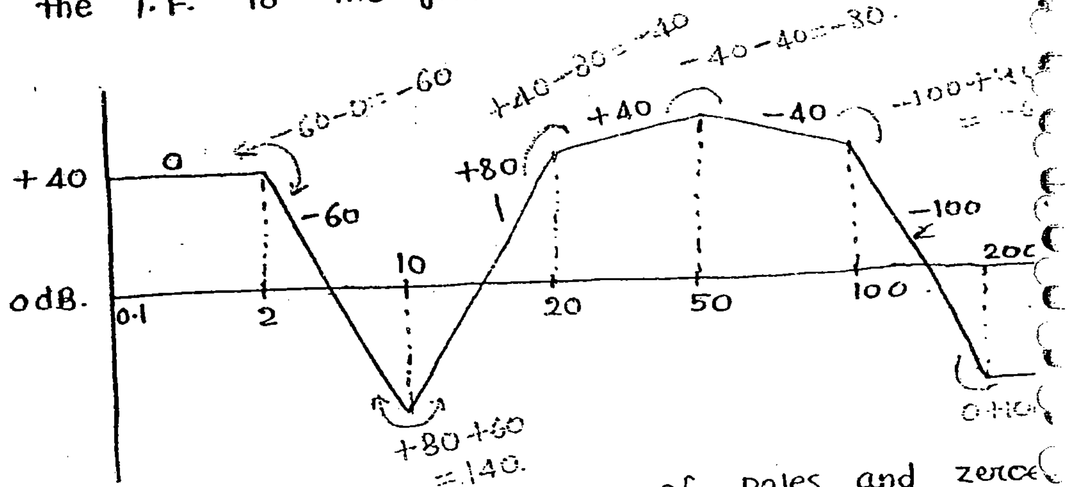
>200
(Included)

75Z
45P
27Z + 3Z = 30Z
+ 500 dB/dec.

around 1000

< 1000 excluded	> 1000 (included)
95P	295P
175Z	175Z
80Z	120P
1600 dB/dec.	−2400 dB/dec.

→ Find the T.F. to the given Asymptotic magnitude plot



Procedure

* observe the initial slope it gives the no. of poles and zeroes at origin
* find the change in slope at each and every CF.
  If change in slope = +ve consider the finite zero.
  If " " = −ve " " " pole.
* Find the K value by using known magnitude and known

$$G(S)H(S) = \frac{K\left(1+\frac{S}{10}\right)^7\left(1+\frac{S}{200}\right)^5}{\left(1+\frac{S}{2}\right)^3\left(1+\frac{S}{20}\right)^2\left(1+\frac{S}{50}\right)^4\left(1+\frac{S}{100}\right)}$$

Initial slope = 0, So no pole and zero at origin.

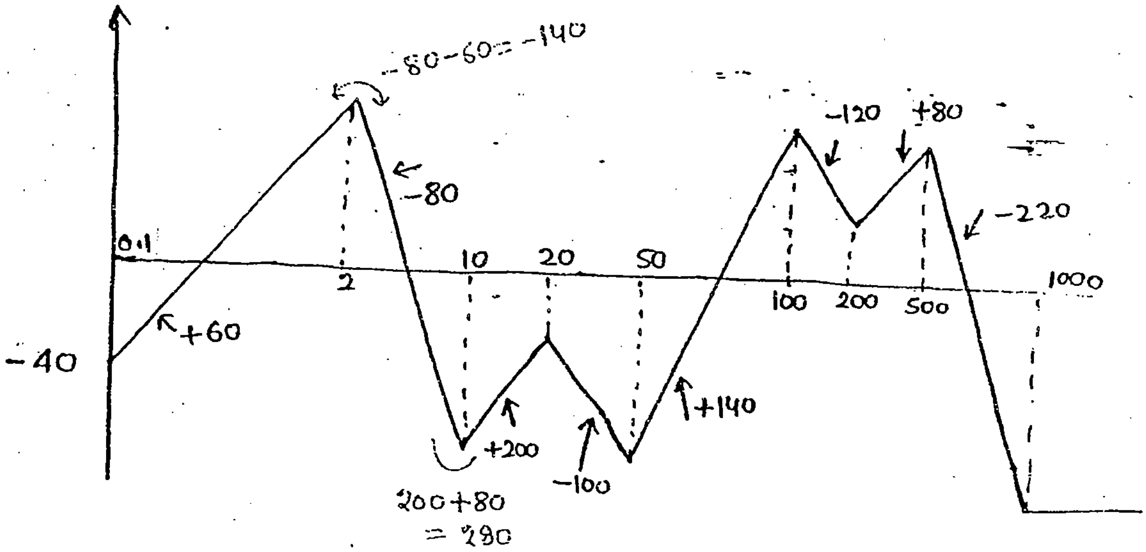
$$40\Big|_{\omega=0.1} = 20\log K - 60\log\sqrt{1+\left(\frac{\omega}{2}\right)^2} + 140\log\sqrt{1+\left(\frac{\omega}{2}\right)^2} + \ldots$$

(neg) (neg)

$$40 = 20\log K$$

$$K = 10^2$$

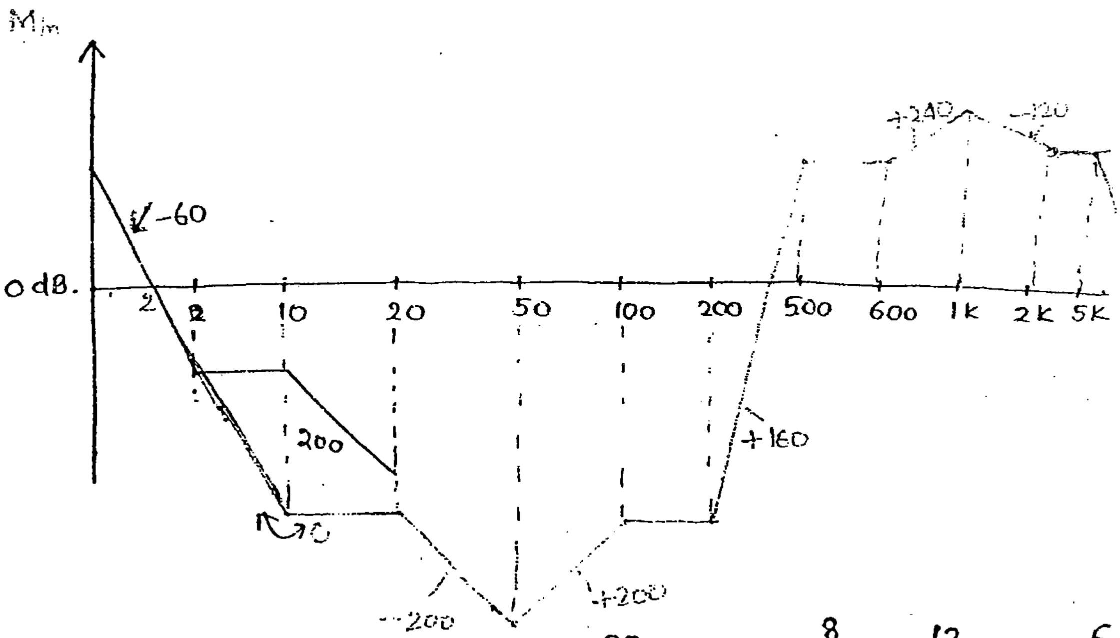
$\boxed{K = 100}$



$$G(S)\,H(S) = \frac{10\,K\,S^3\left(1+S/10\right)^{18}\left(1+S/20\right)^{12}\left(1+S/200\right)^{10}\left(1+S/1000\right)^{1}}{\left(1+\frac{S}{2}\right)^{7}\left(1+S/20\right)^{15}\left(1+S/100\right)^{13}\left(1+S/500\right)^{15}}$$

$$-40\Big|_{\omega=0.1} = 20\log K + 60\log \omega\ ^{0.1}$$

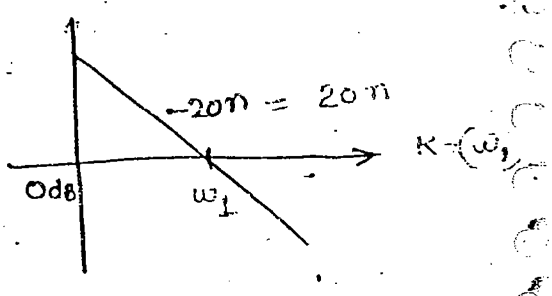
$$\boxed{K = 10}$$

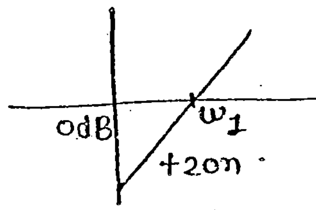


$$G(S)\,H(S) = \frac{K\left(1+S/10\right)^{3}\left(1+S/50\right)^{20}\left(1+S/200\right)^{8}\left(1+S/600\right)^{12}\left(1+S/2K\right)^{6}}{S^3\left(1+S/20\right)^{10}\left(1+S/100\right)^{10}\left(1+S/500\right)^{8}\left(1+\frac{S}{1K}\right)^{18}\left(1+\frac{S}{3K}\right)^{1}}$$

$$0 = 20\log K - 60 \log \omega^{2}$$

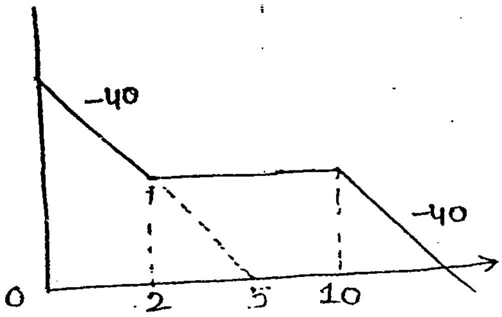
$$\boxed{K=8}$$





$K=(\omega_1)^{-n}$

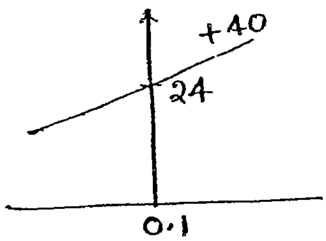
(*)



$K = (5)^2 = 25$

$$\frac{\overset{25}{K}\left(1+ s/2\right)^2}{s^2\left(1+s/10\right)^2}$$

(*)

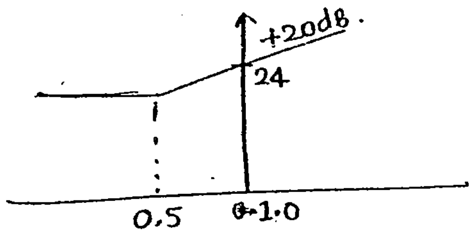


(*) $GH = Ks^2$

$24\big|_{0.1} = Ks^2 \rightarrow 24 = 20\log K + 40 \log$

$K = 10^{3.2} = 1584.8$

(*)

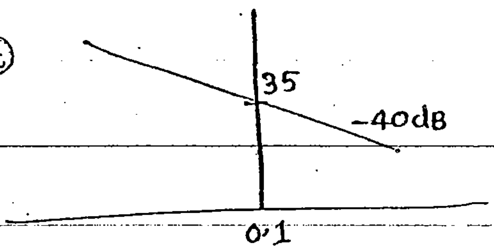


(*) $GH = K\left(1+\frac{s}{0.5}\right)$

$24\big|_{\omega=1.0} = 20\log K + 20\log\sqrt{1+\left(\frac{1}{0.5}\right)^2}$

$24 = 20\log K + 20\log 2$ $\boxed{K=7.9}$

(*)

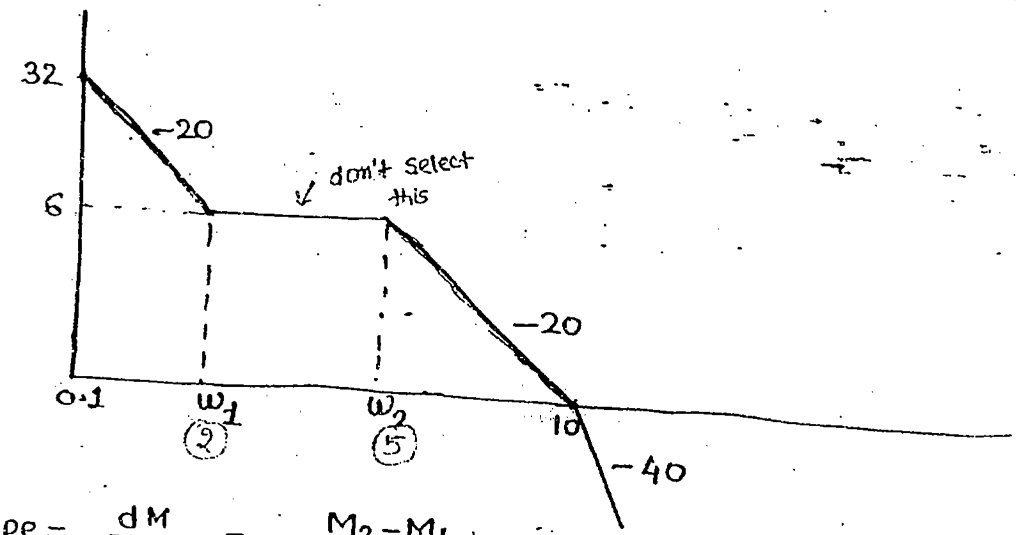


(*) $GH = \frac{K}{s^2}$

$35\big|_{\omega=0.1} = \cancel{20\log K - 40\log\sqrt{}}$

$= 20\log K - 40\log 0.1$

Find ω_1, ω_2 and K for the given magnitude plot



$$\text{Slope} = \frac{dM}{d\log\omega} = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$-20 = \frac{6-32}{\log\omega_1 - \log 0.1}$$

$$\log\omega_1 + 1 = \frac{26}{20} \qquad \omega_1 = 10^{0.3} = 2\ \text{rad/sec}$$

$$-20 = \frac{0 - 6}{\log 10 - \log\omega_2}$$

$$20 - 20\log\omega_2 = 6$$

$$-20\log\omega_2 = -14$$

$$\log\omega_2 = \frac{14}{20}$$

$$\boxed{\omega_2 = 5\ \text{rad/sec}}$$

$$G(s)H(s) = \frac{K\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

$$32\Big|_{\omega=0.1} = 20\log K - 20\log 0.1$$

$$12 = 20\log K$$

$$K = 10^{0.6}$$

$$\boxed{K = 4}$$

$$6\Big|_{\omega=5} = 20\log K - 20\log\omega + 20\log\sqrt{1+\left(\frac{\omega}{2}\right)^2}$$

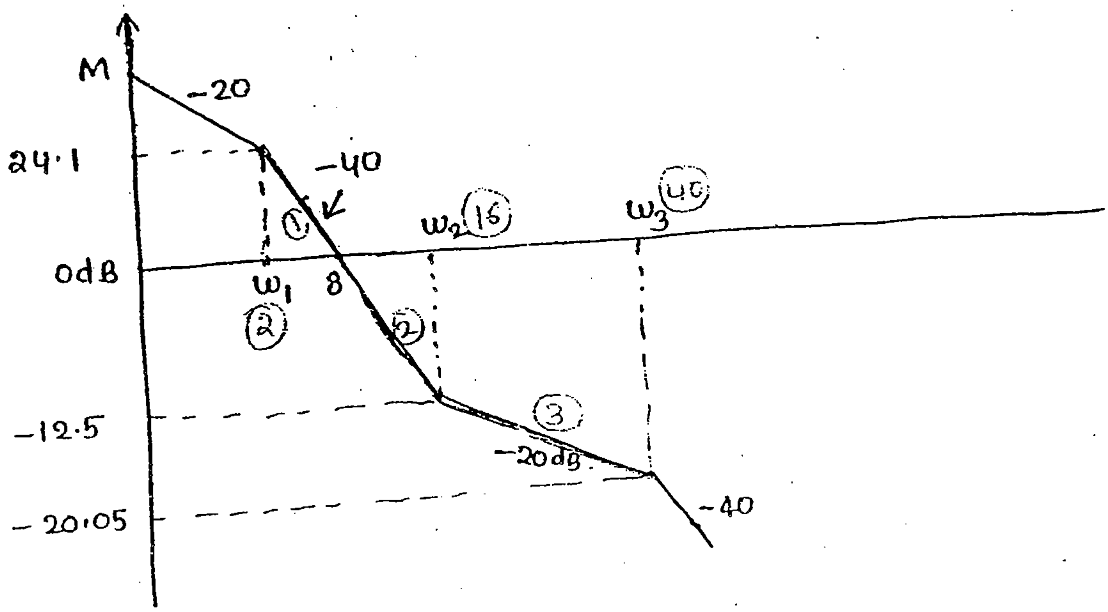
(Neg.)

$$6 = 20\log K - 20\log 5 + 20\log 2.5$$

$$12 = 20\log K$$

$$\boxed{K \simeq 4}$$

(*) Find the magnitude M, $\omega_1, \omega_2, \omega_3$ and T.F.



$$-40 = \frac{0 - 24.1}{\log 8 - \log\omega_1}$$

$$-36.12 + 40\log\omega_1 = -24.1$$

$$40\log\omega_1 = 12.02$$

$$\log\omega_1 = 0.3005$$

$$\boxed{\omega_1 = 2 \text{ rad/sec.}}$$

$$-20 = \frac{-M + 24.1}{\log 2 - \log 1}$$

$$\boxed{M = 30.12 \text{ dB.}}$$

$$-40 = \frac{-12.05 - 0}{\log\omega_2 - \log 8}$$

$$\boxed{\omega_2 = 16 \text{ rad/Sec.}}$$

$$-20 = \frac{-20.05 + 12.05}{\log\omega_3 - \log 16}$$

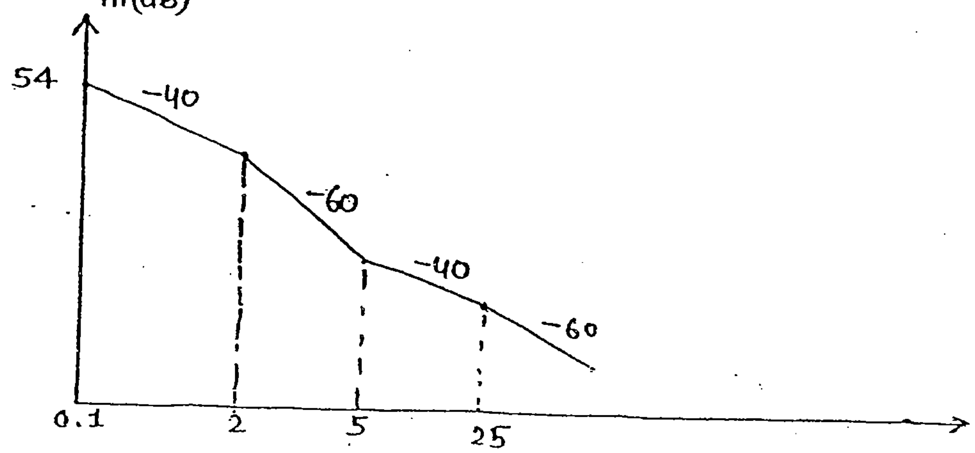
$$\boxed{\omega_3 = 40 \text{ rad/sec}}$$

$$30.12 = \frac{K\left(1+S/16\right)}{S\left(1+S/2\right)\left(1+S/40\right)}$$

$$30.12\Big|_{\omega=1} = 20\log K - 20\log \cancel{\omega}^{-1}$$

$$\boxed{K=32}$$

* The Asympotitic Approx. of the Plot mag. vs freq. plot of min. phase system shown in figure its TF is.



(a) $\dfrac{5\,(s+5)}{s^2\,(s+2)\,(s+25)}$ (b) 10 " — " (c) 40 " — " (d) 50 " —

$$GH\Big|_{\omega=0.1} = \frac{K\left(1+S/5\right)}{S^2\left(1+S/2\right)\left(1+S/25\right)}$$

Time const. form

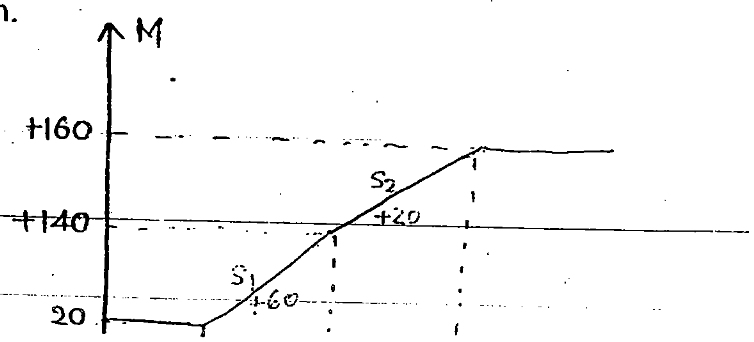
to write Normal form →

$$54 = 20\log K - 40\log 0.1$$

$$K = 10^{0.7} = 5$$

$\dfrac{\cancel{5}\times 2\times 25}{\cancel{5}} = 50$

* The Asymptotic Bode mag. of a min. phase System shown in fig the T.F. of the system.



$$S_1 = \frac{140 - 20}{\log 10 - \log 0.1}$$

$$\boxed{S_1 = 60}$$

$$S_2 = \frac{160 - 140}{\log 100 - \log 10}$$

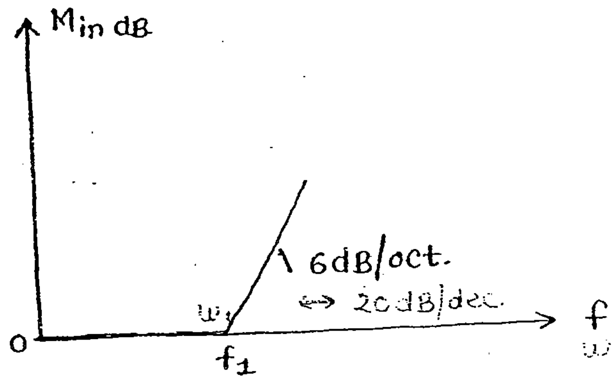
$$S_2 = \frac{20}{1}$$

$$\boxed{S_2 = 20}$$

$$20\ GH\Big|_{0.1} = \frac{10^5\,(K)\left(1 + s/0.1\right)^3}{\left(1 + s/10\right)^2\left(1 + s/100\right)}$$

$$20 = 20 \log K$$

$$K = \frac{10 \times 10^2 \times 100 \times 10^3}{(0.1)^3 \left(1/10\right)^3} = 10^5$$

(*) 

(a) $j\, f/f_1$

(b) $j\, f_1/f$

(c) $\dfrac{1}{1 + j\, f/f_1}$

(d) $\left(1 + j f/f_1\right)$

$$K\left(1 + \frac{s}{\omega_1}\right)$$

$$\frac{K}{1}\left(1 + \frac{j\omega}{\omega_1}\right) = \left(1 + j\frac{2\pi f}{2\pi f_1}\right) = \left(1 + j\, f/f_1\right)$$

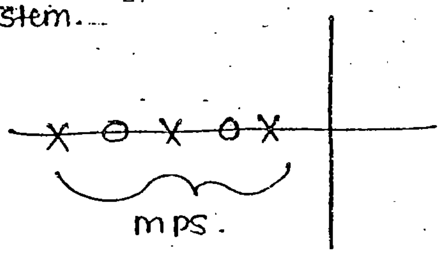
* <u>classification of system</u>

Based on position of zero's But all the poles must be lies in the left Half S-Plane.

→ The Bode plots are valid for minimum phase system.

minimum phase System -

A system in which all the finite poles and finite zeros lies in the left Half S-plane then it is called minimum phase System.



$$mps = \frac{(s+1)}{(s+2)(s+3)}$$

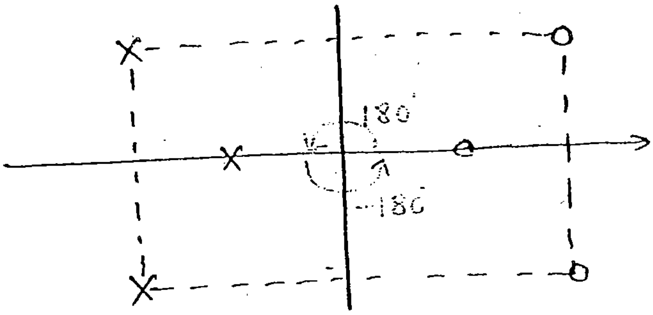
$$\angle\phi = \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan^{-1}\omega/3$$

$$\angle\phi\Big|_{\omega=0} = 0.$$

All pass System

A system in which zero lies in the right Half s-plane which are symmetrical about imagenary axis then poles lies in Left Half S-plane then it is called All pass System.

The All pass system gives the magnitude 1. and ϕ phase angle varies b/w $\pm 180^\circ$

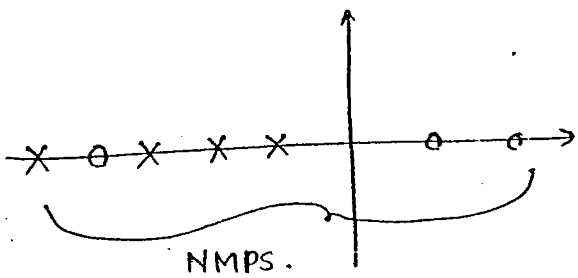


$$ALPS = \frac{(s-1)(s^2-2s+2)}{(s+1)(s^2+2s+2)}$$

$$M = 1 \angle \pm 180^\circ$$

Non-minimum phase System :-

* A system in which some zero's lie in the Right Half S-plane and Remain all poles and zeros lie left Half S-plane then it is called Non-minimum phase system



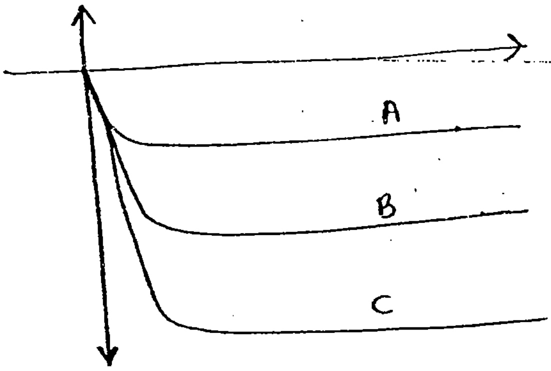
$$NMPS = \frac{(s+1)(s-2)}{(s+3)(s+4)}$$

$$\frac{(s+1)(s+2)}{\quad} \times \frac{s-2}{s+2}$$

$$\boxed{NMPS = MPS \times ALPS}$$

$$\phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}.$$

Q. Identify curves A, B, C in given phase plot



A	-90°	(MPS)
		+
B	-180°	(ALPS)
C	-270°	(NMPS)

*→ Stability Condition –

Stability Conditions are to find the closed loop system stability. The CL system stability given by characteristic equation

i.e. $1 + G(s)H(s) = 0$

→ s Replace by $j\omega$

$1 + G(j\omega)H(j\omega) = 0.$

→ $G(j\omega)H(j\omega) = -1 + j0$

The above equation gives the magnitude and phase

→ $\boxed{\text{magnitude}}$ =

$|G(j\omega)H(j\omega)| = 1$ (linear)

$M_{\text{in dB}} = 20 \log 1 = 0\, dB$ (ω_{gc})

The frequency at which magnitude = 1 in linear and 0 in dB is called the $\boxed{\text{gain cross over frequency.}}$

→ phase

$\angle G(j\omega)H(j\omega) = \angle(-1 + j0)$

$= -180^\circ$ (no. of poles also greater than no. of zeros).

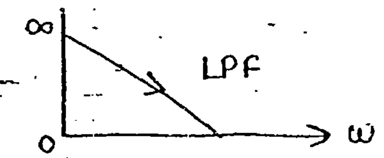
$= \omega_{pc}$

Select a -180°

* for phase cross over freq.

Because control systems are low pass filters more $-ve$ angle than $+ve$

$$\overline{G}H = \frac{K(1+sT_1)}{s^n(1+sT_a)} \qquad P > Z$$



* $\underline{\text{Phase crossover frequency}}$ –
* the freq. at which phase angle is $-180°$ is called the $\boxed{\text{phase crossover frequency}}$

* $\underline{\text{gain margin}} \rightarrow$

$$GM = \frac{1}{\left| G(j\omega) H(j\omega) \right|_{\omega = \omega_{pc}}} \quad \text{(Linear)}$$

$$= -20 \log \left| G(j\omega) H(j\omega) \right|_{\omega = \omega_{pc}} \quad \text{(in dB)}$$

* $\underline{\text{Phase margin}} \rightarrow$

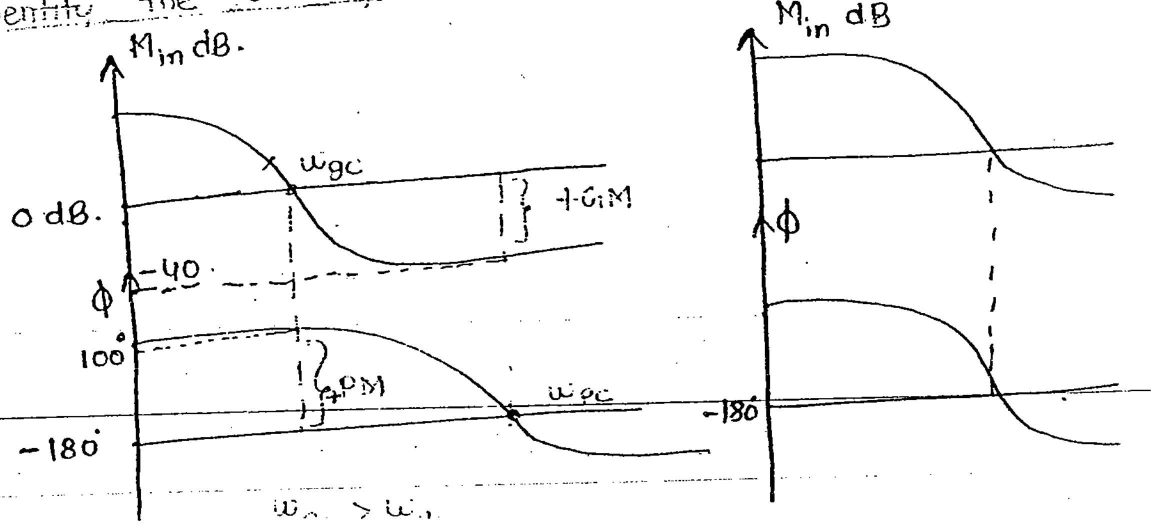
$$PM = 180° + \angle G(j\omega) H(j\omega) \Big|_{\omega = \omega_{gc}}$$

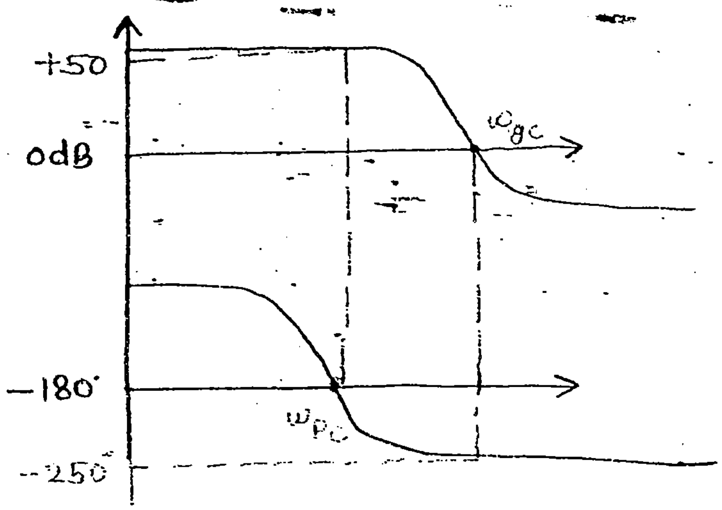
$\omega_{pc} > \omega_{gc} \longrightarrow$ (S) $\left\{ \begin{array}{l} GM = +ve \text{ in dB} \\ GM > 1 \text{ (Linear)} \end{array} \right\}$ and $\underline{PM = +ve}$

$\omega_{pc} = \omega_{gc} \longrightarrow$ m.s. $\left\{ \begin{array}{l} GM = 0 \text{ in dB.} \\ GM = 1 \text{ (Linear)} \end{array} \right\}$ and $\underline{PM = 0}$.

$\omega_{pc} < \omega_{gc} \longrightarrow$ u.s. $\left\{ \begin{array}{l} GM = -ve \text{ (in dB)} \\ GM < 1 \text{ (Linear)} \end{array} \right\}$ and $\underline{PM = -ve}$

* $\underline{\text{Identify the stability in the given Bode plots}}$





① $\omega_{pc} > \omega_{gc}$

$$GM = -(M_{in\,dB})_{\omega_{pc}} = -(-40\,dB) = +40\,dB.$$

$$PM = 180^\circ + \angle GH\Big|_{\omega_{gc}} = 180^\circ - 100 = 180^\circ$$

② $\omega_{pc} = \omega_{gc}$

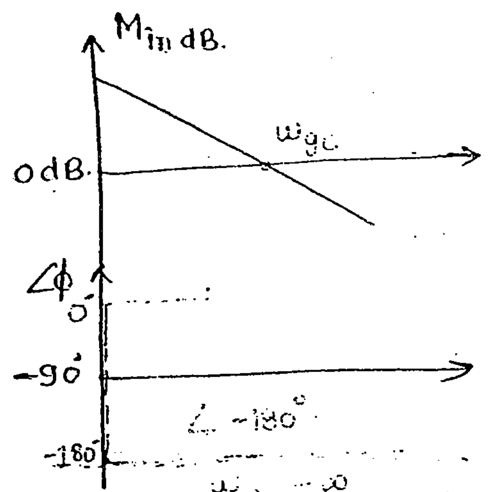
$GM = +0\,dB.$

$PM = 180^\circ - 180^\circ = 0^\circ$

③ $\omega_{pc} < \omega_{gc}$

$$\left.\begin{aligned} GM &= -(50\,dB) \\ PM &= 180^\circ - 250^\circ = -70^\circ \end{aligned}\right\}$$ Both −ve so system → $\boxed{\text{unstable}}$

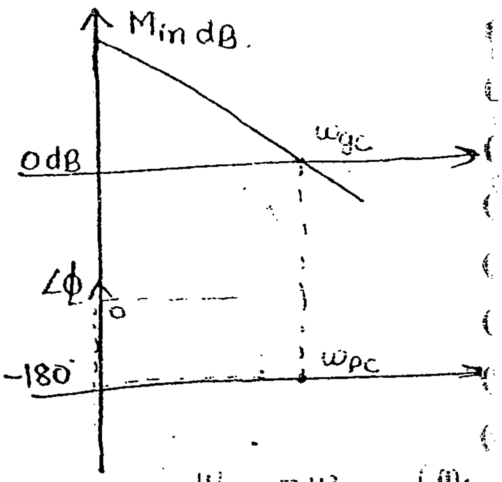
Q.



$\omega_{pc} = \infty$.

$\omega_{pc} >> \omega_{gc}$ →(S)

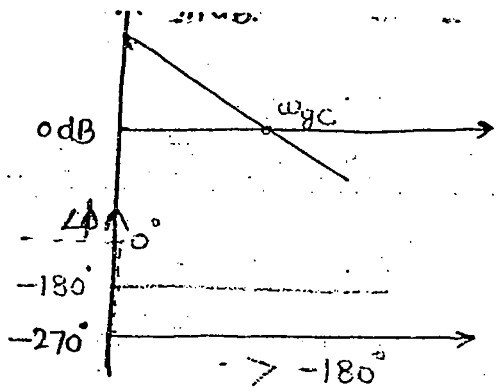
GH = 1/S   CLTF = $\frac{1}{S+1}$



$\omega_{pc} = \omega_{gc}$ (m...

GH = $1/s^2$ CLTF = ...

$\times + j1$



$\longrightarrow \omega_{pc} = 0.$

$\omega_{pc} << \omega_{gc} \longrightarrow$ (U.S.)

$GH = \frac{1}{s^3}$ $\quad$ $CL = \underbrace{\frac{1}{s^3+1}}_{U.S.}$

* whenever the plot or T.F. maintains less negative than $-180°$ at all the frequency Range then $\omega_{pc} = \infty$. in this case, the system is stable because $\omega_{pc} >> \omega_{gc}$

* whenever the plot or T.F. maintains the Phase angle of $-180°$ at all the frequency Range then value of ω_{pc} decided by ω_{gc} in this case, ω_{pc} must equal to ω_{gc}. and System becomes the M.S.

* whenever plot or T.F. maintains more $-ve$ than $-180°$ at all the frequency range then $\omega_{pc} = 0$. In this case the system is unstable because $\omega_{pc} << \omega_{gc}$

* Complex Bode plot :-

n - Complex Poles

$$G(s)H(s) = \left[\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right]^n$$

n - complex zero

$$G(s)H(s) = \left[\frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2}\right]$$

S - Replace by $j\omega$

$$G(j\omega)H(j\omega) = \left(\frac{\omega_n^2}{-\omega^2+j2\zeta\omega_n\omega+\omega_n^2}\right)^n$$

$$= \left[\frac{1}{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}+j2\zeta\left(\frac{\omega}{\omega_n}\right)}\right]^n$$

$$M = \left[\frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}\right]^n$$

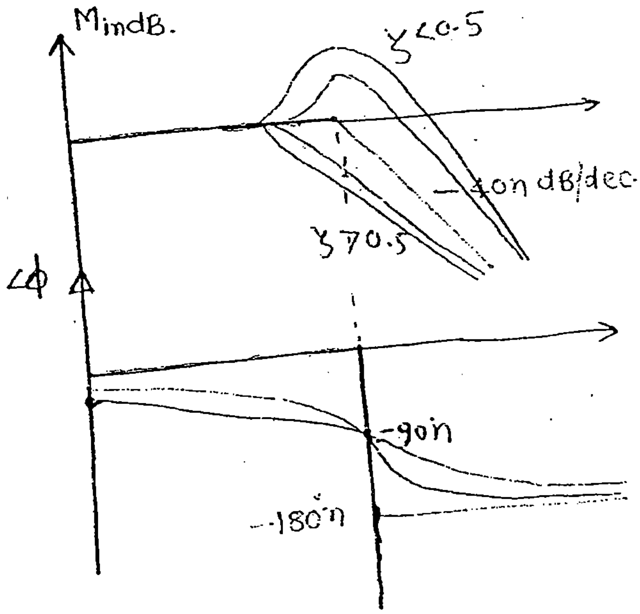
$$M_{\text{in dB actual}} = -20n \log \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}$$

$$\phi_{actual} = -n \tan^{-1}\left(\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right)$$

Asymptotic analysis

$\omega_n \rightarrow$ corner freq.

	Slope	$\angle\phi$
< CF	0	$0°$
> CF	$-40n$	$-180°n$



$M_{\text{correction at CF}}$ $(\omega=\omega_n)$

$= -20n \log(2\zeta)$

$\phi_{\text{correction at CF (at } \omega=\omega_n)} = -90°n.$

- correction at corner freq. depends on ζ in the mag. plot
- whereas the phase plot the correction at C.F. is $-90^\circ n$ and constant
- other than C.F. the correction depend on ζ and ω_n.

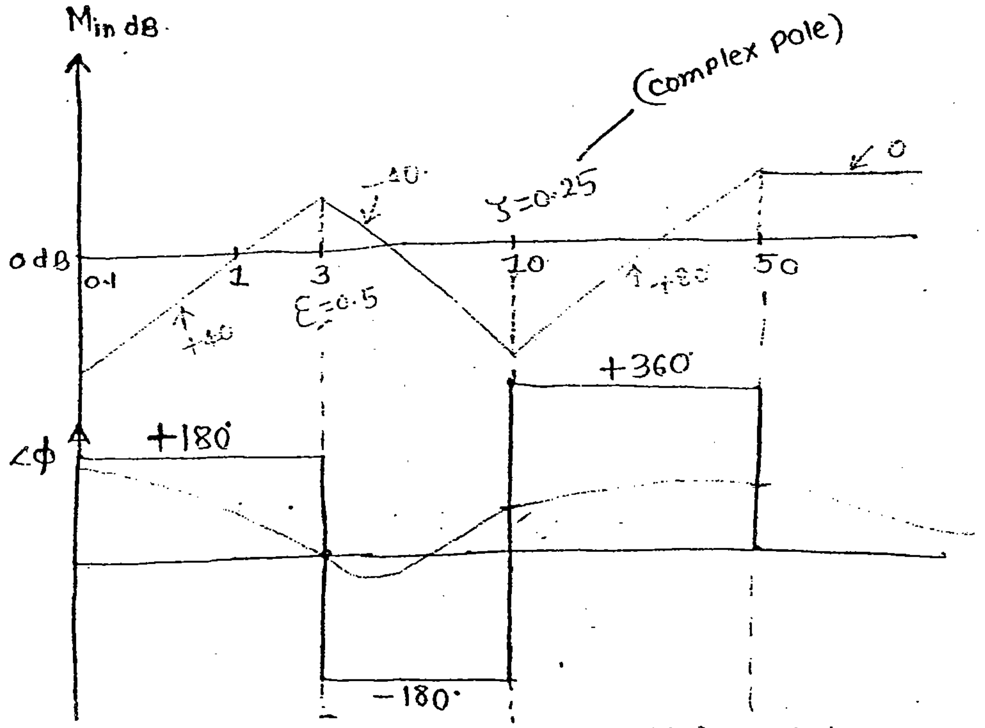
Draw the Bode plot

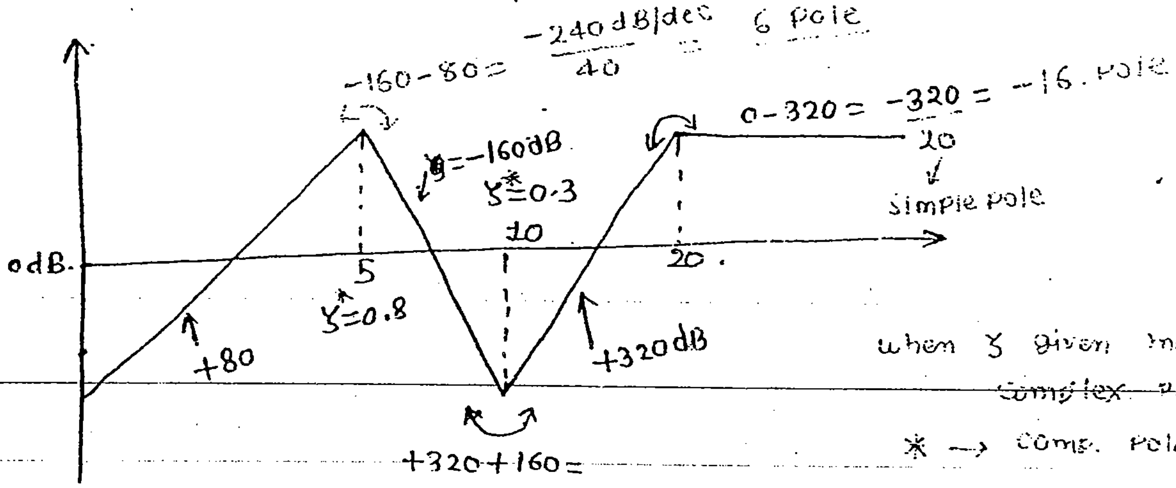
$$\frac{2\zeta}{\omega_n} = \frac{1}{20} \qquad \boxed{\zeta = 0.25}$$

Started from $-40 \rightarrow +40$ dB.

$$G(s)H(s) = \frac{s^2\left(1+\frac{s}{20}+\frac{s^2}{100}\right)^3}{\left(1+\frac{s}{3}+\frac{s^2}{9}\right)^2\left(1+\frac{s}{50}\right)^4} \qquad \omega_n = 10$$

$$\omega_n = 3 \qquad \frac{2\zeta}{\omega_n} = \frac{1}{3} \qquad \boxed{\zeta = 0.5}$$





when ζ given means ~~complex pole~~

$* \rightarrow$ comp. pole

$$T\cdot F\Big|_{\omega=0.1} = K\cdot s^4\left(\frac{25}{s^2+8s+25}\right)^6\left(\frac{s^2+6s+100}{100}\right)\left(1+\frac{s}{2}\right)$$

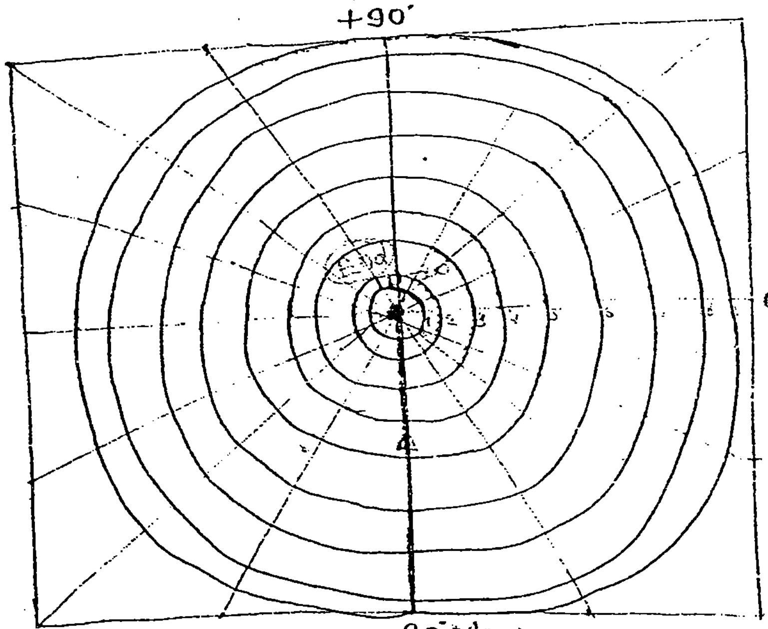
$$-40 = 20\log K + 80\log 0.1$$

$$40 = 20\log K$$

$$\boxed{K=100}$$

Polar plot –

Purpose
* to draw the frequency response of open loop transfer function.
* to find the closed loop system stability.
* to find the gain crossover freq., PM, ω_{pc}
* to find the gain margin.
* the polar plots are used in the Nyquist plot to find the closed loop system stability. (to draw the N.P.)
* the polar plots are not a complete freq. response plot, the complete freq. Response plot is Nyquist plot.
* Polar plot is nothing but magnitude vs phase plot.



$G(s)H(s) = 1/s$. [graph]

(1) $G(S)H(S) = \dfrac{1}{S+1}$

S Replaced by $j\omega$.

$$M = \frac{1}{\sqrt{\omega^2+1}}$$

$$\angle\phi = -\tan^{-1}\omega.$$

$\omega = 0$ $\quad M_1 = 1$ $\quad \angle\phi_1 = 0$

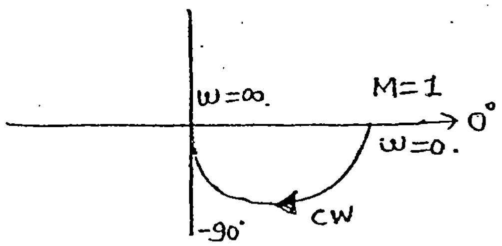
$\omega = \infty$ $\quad M_2 = 0$ $\quad \angle\phi_2 = -90^\circ$

Ending ~~Starting~~ direction $= \phi_1 - \phi_2$

$= 90^\circ$

$= +ve.$

Ending direction = CW.



(3) $\dfrac{0.5}{(1+s)(1+0.5s)}$

$$\omega = \frac{1}{\sqrt{1\times 0.5}} = \sqrt{2}$$

$$M = \frac{0.5\sqrt{1\times 0.5}}{1+0.5}$$

$$= \frac{0.5\sqrt{0.5}}{1.5}$$

(2) $G(S)H(S) = \dfrac{1}{(S+1)(S+2)}$

$$M = \frac{1}{\sqrt{(\omega^2+1)(\omega^2+4)}}$$

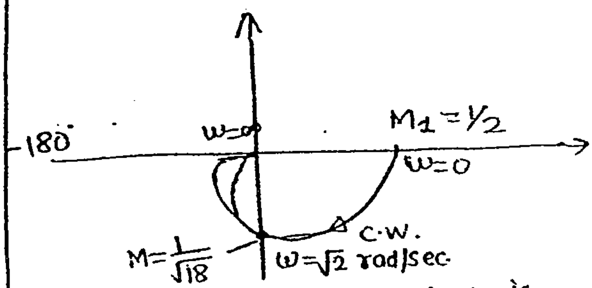
$$\angle\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$\omega = 0,$ $\quad M_1 = 1/2$ $\quad \angle\phi_1 = 0$

$\omega = \infty$ $\quad M_2 = 0$ $\quad \angle\phi_2 = -$

E.D. $= +180^\circ$ (CW)

S.D = (finite pole) so CW



The intersection point is nothing but magnitude but not the frequency.

Intersection point with -90°

$$\angle GH = -90^\circ$$

$$-90^\circ = -\tan^{-1}\frac{3\omega/2}{1-\omega^2/2}$$

$$\frac{1}{0} = \frac{3\omega/2}{1-\omega^2/2}$$

$\boxed{\omega = \sqrt{2}}$ Rad/sec.

$$M\Big|_{\omega=\sqrt{2}} \Longrightarrow 1/\sqrt{18}.$$

Intersection point $\left(0, -\dfrac{j}{\sqrt{18}}\right)$

(3) if $\quad GH = \dfrac{K}{(ST_1+1)(ST_2+1)}$

then

$$\omega = \frac{1}{\sqrt{T_1T_2}}$$

$$M = \frac{K\sqrt{T_1T_2}}{T_1+T_2}$$

Draw the polar plot

$$G(S)\,H(S) = 1/S.$$

→ S Replaced by $j\omega$

$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega} \rightarrow \boxed{M = 1/\omega}$$

$$\angle\phi = \frac{\angle 1}{\angle j\omega} = -90^\circ$$

ω	M	$\angle\phi$
0	∞	-90°
1	1	-90°
2	0.5	-90°
5	0.2	-90°
10	0.1	-90°
∞	0	-90°

0 to 1

Q. $$G(S)\,H(S) = \frac{1}{S\tau + 1}$$

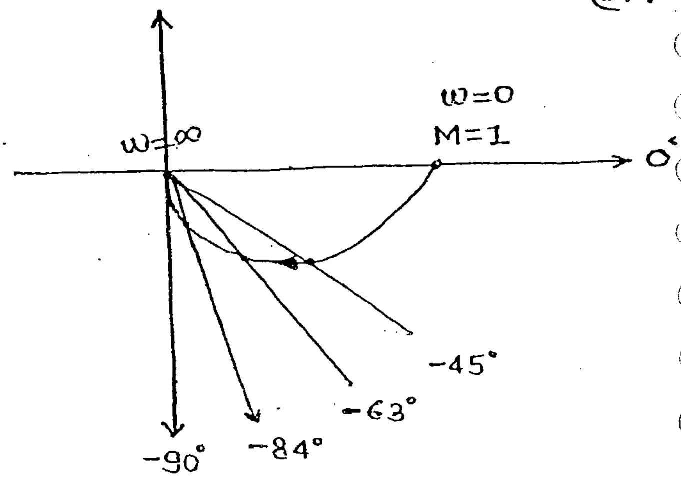
$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega\tau + 1}$$

$$\boxed{M = \frac{1}{\sqrt{(\omega\tau)^2 + 1}}}$$

$$\angle\phi = \frac{\angle 1}{\angle \tan^{-1}\frac{\omega\tau}{1}} = -\tan^{-1}(\omega\tau)$$

(LPF)

ω	M	ϕ
0	1	0°
$1/\tau$	0.707	-45°
$2/\tau$	0.44	-63.43°
$\frac{10}{\tau}$	0.1	-84°
∞	0	-90°



Q. * $$G(S)\,H(S) = \frac{S+1}{S+10}$$ (Lead compensator) or HPF

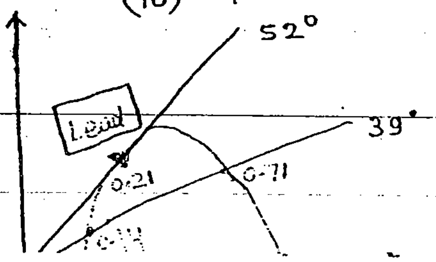
S → Replaced by $j\omega$.

$$G(j\omega)\,H(j\omega) = \frac{j\omega + 1}{j\omega + 10}$$

$$M = \frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}}$$

$$\angle\phi = \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	M	$\angle\phi$
0	0.1	0°
1	0.14	39°
2	0.21	52.1
5	0.45	52°
10	0.71	39°
∞	1	0°



* $$G(S)\,H(S) = \frac{S+10}{S+1}$$

$$M = \frac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+1}}$$ (Lag comp. or LPF).

$$\angle\phi = \tan^{-1}\frac{\omega}{10} - \tan^{-1}\omega$$

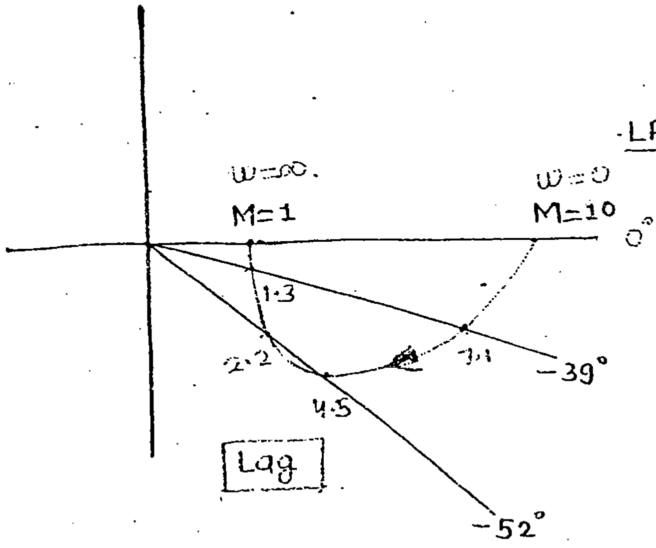
ω	M	ϕ
0	10	0°
1	7.1	-39°
2	4.5	-52°
5	2.2	-52°
10	1.3	-39°
∞	1	0°

HPF → If $M|_{\omega=0} < M|_{\omega=\infty}$

then follow Std. procedu

LPF → $M|_{\omega=0} > M|_{\omega=\infty}$

then follow Std. procedu



① If the T.F. maintains, magnitude at $\omega=0$ is less than the magnitude at $\omega=\infty$ then get the solution by using the Standard procedure like lead compensator (HPF)

Procedure →

This procedure is valid only when the starting magnitude is the greater than the ending magnitude like low pass filter (lag compensator).

Step-1 Find the magnitude and phase at $\omega=0$

Step-2 Find the mag. and phase at $\omega=\infty$

Step-3 Find the ending direction.

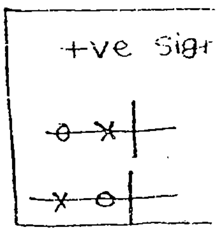
at $\omega=0$ $M_1\angle\phi_1$

$\omega=\infty$ $M_2\angle\phi_2$

Ending direction $= \phi_1-\phi_2$

$=$ +ve (CW)

$=$ −ve (ACW)



The starting direction is valid only to the T.F. which are cons +ve sign term

If the finite pole is near to the Imagenary axis then Startin direction − CW.

If the finite zero is near to the Imagenary axis then sta dir ACW.

$$GH = \frac{1}{(s+1)(s+2)(s+3)}$$

$$M = \frac{1}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}}$$

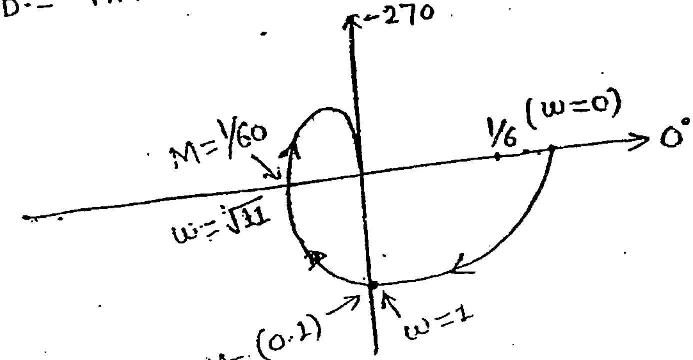
$$\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

at $\omega = 0$ $\quad M = 1/6 \quad \angle\phi = 0^\circ$

at $\omega = \infty$ $\quad M = 0 \quad \angle\phi = -270^\circ$

E.D. $= \phi_1 - \phi_2 = 270^\circ$ (+ve) (cw).

S.D. = finite pole = cw.



I.P. with -90°

$$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

$$+90^\circ = \tan^{-1}\omega + \tan^{-1}\left(\frac{5\omega}{6-\omega^2}\right)$$

$$90^\circ = \tan^{-1}\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \omega \times \frac{5\omega}{6-\omega^2}}$$

$$1 - \frac{5\omega^2}{6-\omega^2} = 0$$

$$5\omega^2 = 6 - \omega^2$$

$$6\omega^2 = 6$$

$$\boxed{\omega = 1 \text{ rad/sec}}$$

$M = 1/6$ I.P. with -180°

$$\angle GH = -180^\circ$$

$$180^\circ = \tan^{-1}\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \omega\left(\frac{5\omega}{6-\omega^2}\right)}$$

$$\boxed{\omega = \sqrt{11}}$$

* addition of each finite pole, shift the ending angle by -90° in the clockwise direction

$$\omega = \frac{1}{\sqrt{T_1T_2 + T_2T_3 + T_3T_1}}$$

$$\omega = \frac{T_1 + T_2 + T_3}{T_1 T_2 T_3}$$

Q. $GH = \frac{1}{s(s+1)}$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

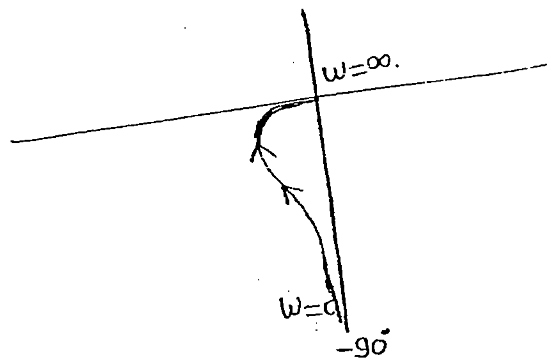
$$\angle\phi = -90^\circ - \tan^{-1}\omega.$$

$\omega = 0 \quad M = \infty \quad \angle\phi = -90^\circ$

$\omega = \infty \quad M = 0. \quad \angle\phi = -180^\circ$

ED = CW

SD = CW.



$$GH = \frac{1}{s^2(s+1)}$$

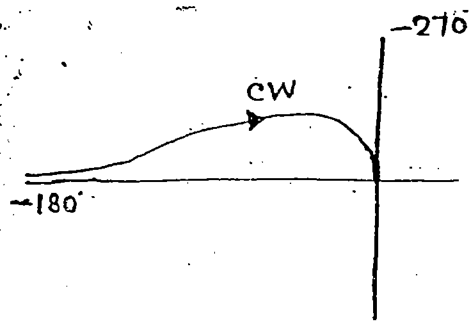
$$M = \frac{1}{-\omega^2\sqrt{\omega^2+1}} \qquad \angle\phi = -180^\circ$$

$\omega = 0 \quad M = \infty \quad \phi = -180^\circ$

$\omega = \infty \quad M = 0 \quad \phi = \angle -270^\circ$

E.D. $\phi_1 - \phi_2 = +90^\circ$ (CW)

S.D. — finite pole — CW



$$GH = \frac{1}{s^3(s+1)}$$

$$M = \frac{1}{\omega^3\sqrt{\omega^2+1}}$$

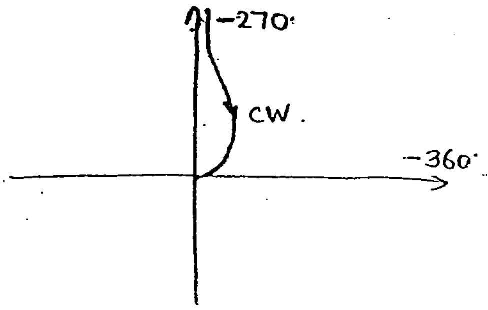
$$\angle\phi = -270^\circ - \tan^{-1}\omega.$$

at $\omega = 0$ $\quad \infty \angle -270^\circ$

$\omega = \infty$ $\quad 0 \angle -360^\circ$

E.D. → CW.

S.D → Finite pole → CW.



The addition of each pole at origin shift total plot by -90° in the clockwise direction.

Q. $\quad GH = \frac{s+1}{s^3}.$

$$M = \frac{\sqrt{1+\omega^2}}{\omega^3}$$

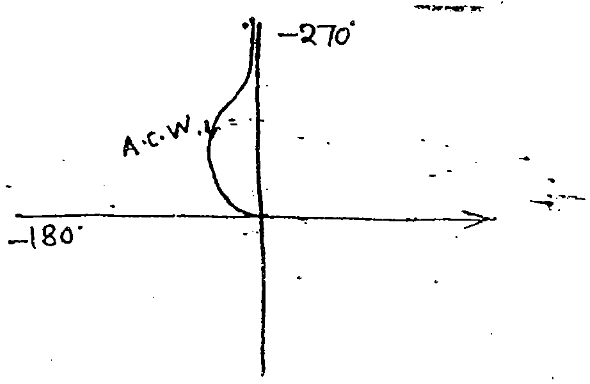
$$\angle\phi = -270^\circ + \tan^{-1}\omega.$$

$\omega = 0$ $\quad \infty \angle -270^\circ$

$\omega = \infty$ $\quad 0 \angle -180^\circ$

E.D. = ACW.

S.D. = ACW (due to finite zero)



Q. = $\frac{(s+1)(s+2)}{s^3} = GH$

$$M = \frac{\sqrt{(\omega^2+1)(\omega^2+4)}}{\omega^3}$$

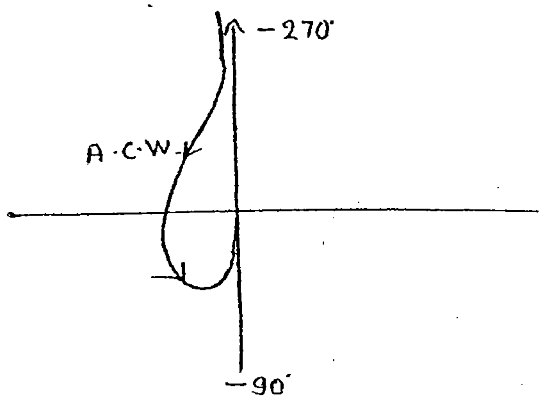
$$\angle\phi = -270^\circ - \tan^{-1}\omega - \tan^{-1}\omega/2$$

at $\omega = 0$ $\quad M = \infty \quad \angle\phi = -27$

$\omega = \infty$ $\quad M = 0 \quad \angle\phi = -9c$

E.D. = ACW.

S.D. = Finite zero (ACW)



Q.

$$GH = \frac{(s+1)(s+2)(s+3)}{s^3}$$

$$M = \frac{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}}{\omega^3}$$

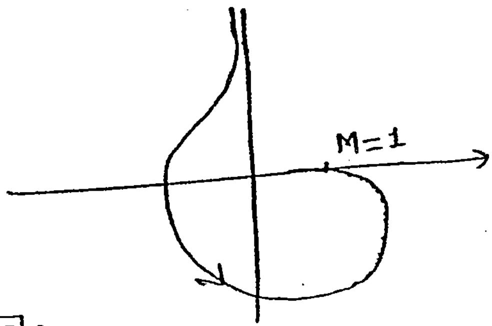
$$\angle\phi = -270^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3}$$

$\omega = 0$ $\quad \infty \angle -270^\circ$

$\omega = \infty$ $\quad 1 \angle 0^\circ$

E.D. → A.C.W.

S.D. → finite zero → A.C.W.



NOTE → the addition of each finite zero shift the ending angle by $+90^\circ$ in the Anticlockwise direction.

Q. ⇒ $GH = S$

Note - whenever T.F. consist the poles or zero's at origin then the polar plot is nothing but angle line

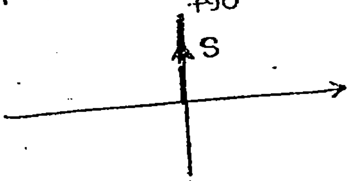
$M = \omega$ $\quad \angle\phi = 90^\circ$

$\omega = 0$ $\quad 0 \angle +90^\circ$

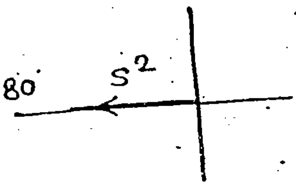
$\omega = \infty$ $\quad \infty \angle +90^\circ$

No { Starting direction
No { Ending direction

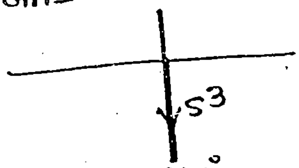
It is Straight line along $+90^\circ$



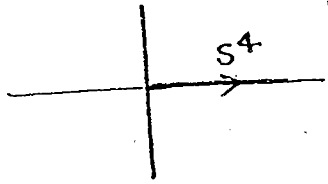
$GH = S^2$



$GH = S^3$



$GH = S^4$



* The addition of each zero at origin shift the total plot by $+90^\circ$ in the ACW direction

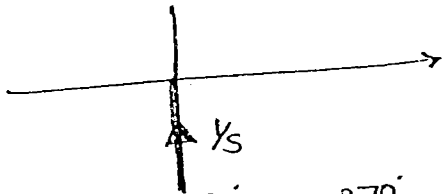
Q. $GH = 1/s$

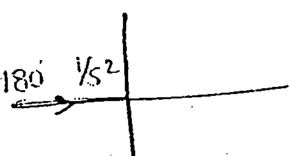
$M = \frac{1}{\omega}$ $\quad \angle\phi = -90^\circ$

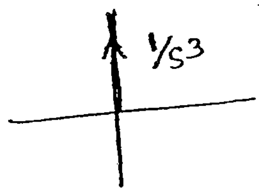
$\omega = 0$ $\quad \infty \angle -90^\circ$

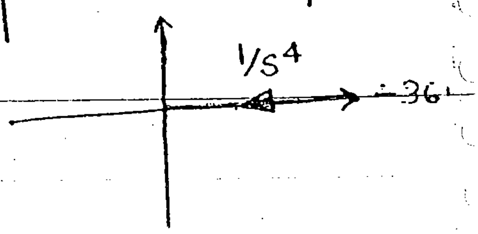
$\omega = \infty$ $\quad 0 \angle -90^\circ$

{ No starting dir^n
{ No Ending dir^n.









$$GH = \frac{s+1}{s^3(s+2)}$$

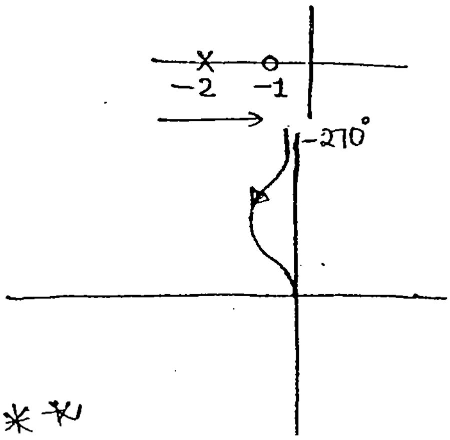
$$M = \frac{\sqrt{\omega^2+1}}{\omega^3\sqrt{\omega^2+4}}$$

$$\angle\phi = -270^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

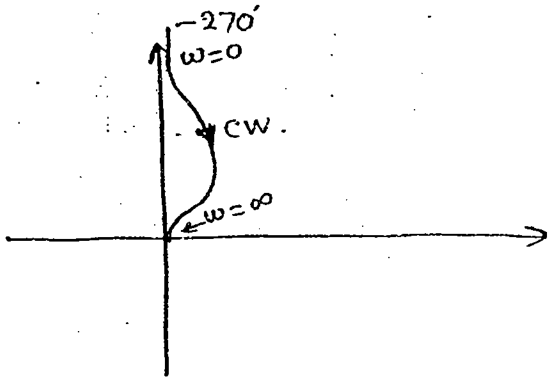
$\omega = 0$	$M = \infty$	$\angle\phi = -270^\circ$
$\omega = \infty$	$M = 0$	$\angle\phi = -270^\circ$

E.D.

S.D. $\longrightarrow$ (Finite zero) ACW.



E.D. X.



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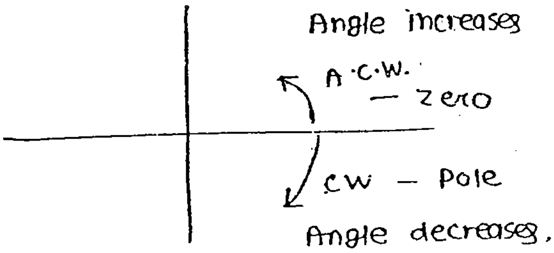
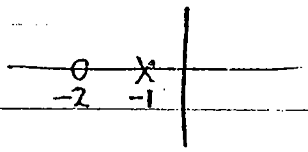


⊛ $$GH = \frac{s+2}{s^3(s+1)}$$

$$M = \frac{\sqrt{\omega^2+4}}{\omega^3\sqrt{\omega^2+1}}$$

$$\phi = -270^\circ + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\omega$$

$\omega = 0$	$M = \infty$	$\phi = -270^\circ$
$\omega = \infty$	$M = 0$	$\phi = -270^\circ$



finite pole = cw.

⊛ $$GH = \frac{(s+1)}{s^2(s+2)(s+3)}$$

$$M = \frac{\sqrt{\omega^2+1}}{\omega^2\sqrt{\omega^2+4}\sqrt{\omega^2+9}}$$

$$\angle\phi = \tan^{-1}\omega - 180^\circ - \tan^{-1}\frac{\omega}{2} - \tan$$

at Low freq. = ($\angle$ zero) > ($\angle$ pole)

at High freq. = ($\angle$ pole) > ($\angle$ zero)

at $\omega = 0$ $M = \infty$ $\angle\phi = -18$

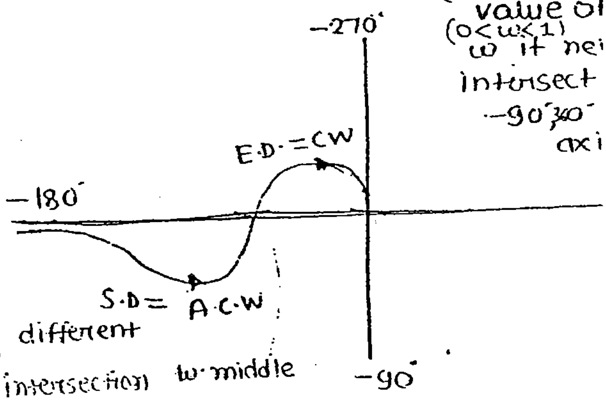
$\omega = \infty$ $M = 0$ $\angle\phi = -27$

Plot have many intersection poi

S.D. Finite zero $\rightarrow$ ACW.

E.D. $\rightarrow$ CW.

(For any value of ($0<\omega<1$) ω it nei intersect $-90°, 360°$ axi



S.D. } different
E.D. }
then intersection b/w middle

<u>intersection with -180°</u>

$$-180 = -180^\circ + \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan$$

$$6 - \omega^2 = 5$$

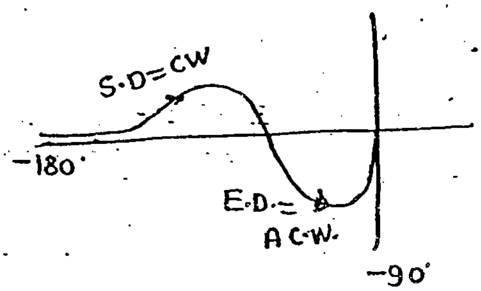
$$\boxed{\omega = 1 \text{ rad/sec.}}$$

Q.

$$GH = \frac{S+3}{S^2(S+1)(S+2)}$$

$$M = \frac{\sqrt{\omega^2+9}}{\omega^2\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}$$

$$\angle\phi = -180^\circ + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$



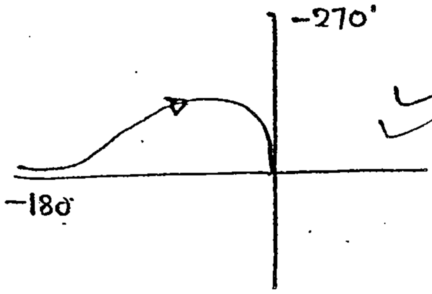
at

$\omega=0$ $\quad M=\infty \quad \angle\phi=-180^\circ$

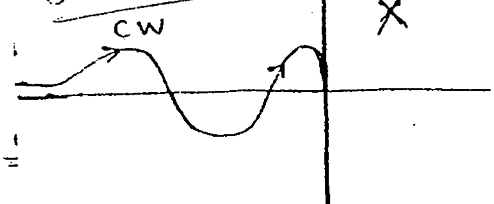
$\omega=\infty$ $\quad M=0 \quad \angle\phi=-270^\circ$

E.D $\longrightarrow$ CW

S.D $\longrightarrow$ finite pole $\rightarrow$ C.W.



So not valid



I.P. with -180°

$\boxed{\omega = \pm j\sqrt{?}}$ invalid

never be imaginary.

$$GH = \frac{(S+1)(S+2)}{S^2(S+3)}$$

$$M = \frac{\sqrt{\omega^2+4}\,\sqrt{\omega^2+1}}{\omega^2\sqrt{\omega^2+9}}$$

$$\phi = -180^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} - \tan^{-1}$$

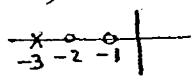
at

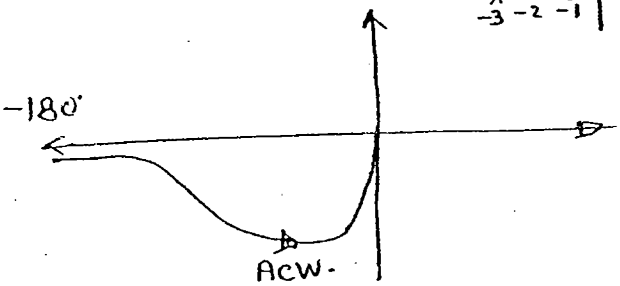
$\omega=0$ $\quad M=\infty \quad \angle -180^\circ$

$\omega=\infty$ $\quad M=0 \quad \angle -90^\circ$

E.D = A.CW.

S.D. = (Finite zero) ACW.





Q.

$$GH = \frac{(S+2)(S+3)}{S^2(S+1)}$$

$$M = \frac{\sqrt{(\omega^2+4)(\omega^2+9)}}{\omega^2\sqrt{\omega^2+1}}$$

$$\angle\phi = -180^\circ + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega.$$

$\omega=0$ $\quad M=\infty \quad \angle -180^\circ$

$\omega=\infty$ $\quad M=0 \quad \angle -90^\circ$

ED = (-ve) $\rightarrow$ ACW.

SD = CW. (finite pole).

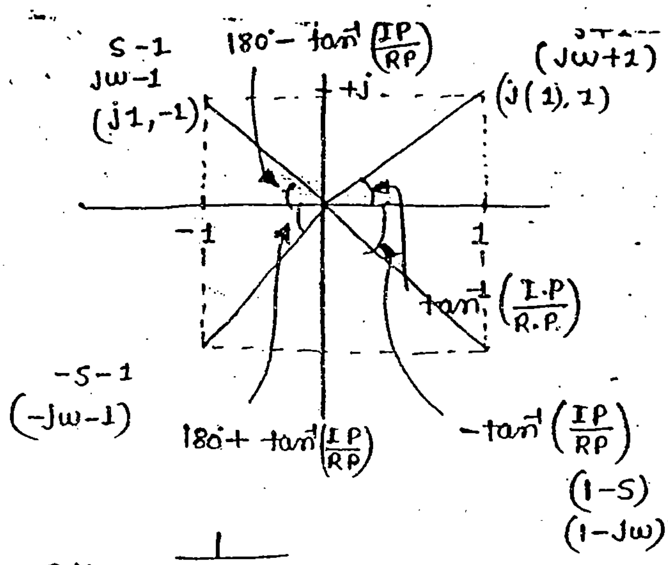
* Draw the polar plot

$$GH = \frac{1}{S(S+1)}$$

$$GH = \frac{1}{S(S-1)}$$

$$GH = \frac{1}{S(-S-1)}$$

$$GH = \frac{1}{S(1-S)}$$



① $GH = \dfrac{1}{S(S+1)}$

$\angle\phi = -90° - \tan^{-1}\omega$

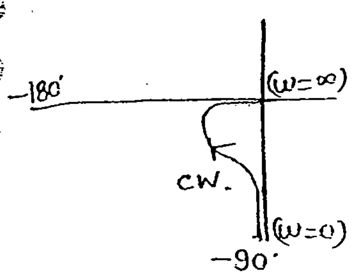
$M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$

$\omega = 0 \qquad \infty\angle -90°$

$\omega = \infty \qquad 0\angle -180°$

E.D. $\longrightarrow$ CW.

S.D. $\longrightarrow$ CW.



② $GH(j\omega) = \dfrac{1}{S(S-1)}$

$M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$

$\angle\phi = -90° - 180° + \tan^{-1}\omega$

$= -270 + \tan^{-1}\omega$

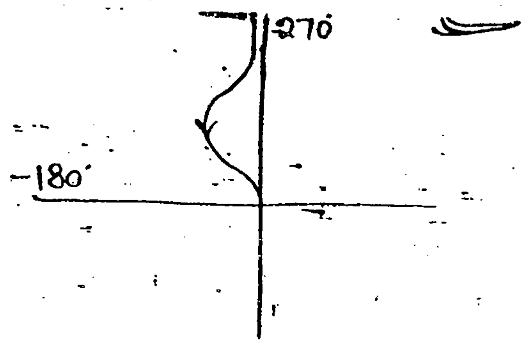
$\omega = 0 \qquad \infty\angle -270°$

$\omega = \infty \qquad 0\angle -180°$

E.D. = ACW.

S.D. = not required

(-ve sign in T.F.)



③ $GH = \dfrac{1}{S(-S-1)}$

$\phi = -90° - (180° + \tan^{-1}\omega)$

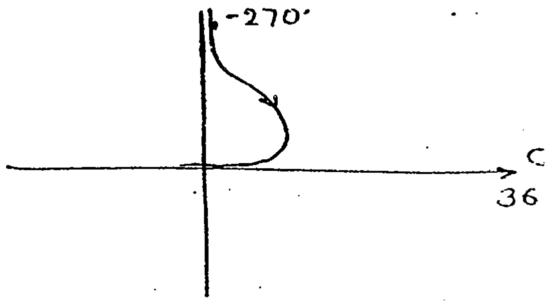
$= -270 - \tan^{-1}\omega.$

$\omega = 0 \longrightarrow \infty\angle -270°$

$\omega = \infty \longrightarrow 0\angle -360°$

ED $\longrightarrow$ CW

SD $\longrightarrow$ Not required.



④ $GH = \dfrac{1}{S(1-S)}$

$\phi = -90° - (-\tan^{-1}\omega)$

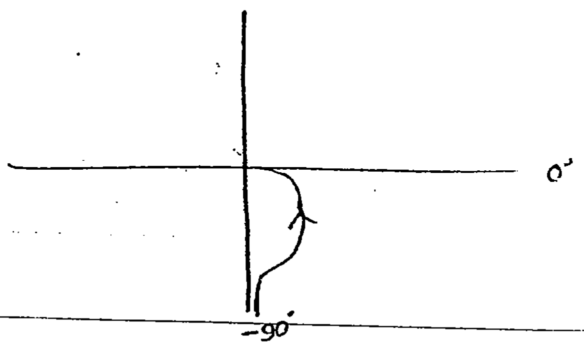
$= -90° + \tan^{-1}\omega$

$\omega = 0 \qquad \infty\angle -90°$

$\omega = \infty \qquad 0\angle 0°$

E.D. $\longrightarrow$ ACW.

S.D $\longrightarrow$ Not required



Q. $GH = \frac{(S+2)}{(S+1)(S-1)}$,

$$|G(j\omega)H(j\omega)| = \frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}\ \sqrt{\omega^2}}$$

$$\angle\phi = \tan^{-1}\frac{\omega}{2} - \left(180^\circ - \tan^{-1}\omega\right) + \tan^{-1}\frac{\omega}{2}$$

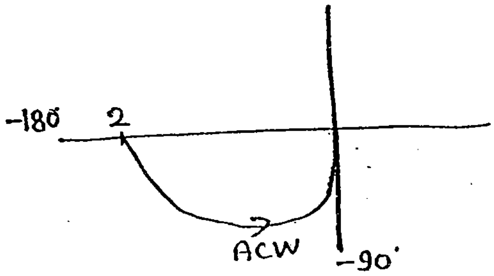
$$= -180^\circ + \tan^{-1}\frac{\omega}{2}$$

$\omega = 0$ $\quad 2\angle -180^\circ$

$\omega = \infty$ $\quad 0\angle -90^\circ$

E.D. → A.C.W.

S.D. → X



$$\phi = \tan^{-1}\frac{\omega}{2} - (180^\circ - \tan^{-1}$$

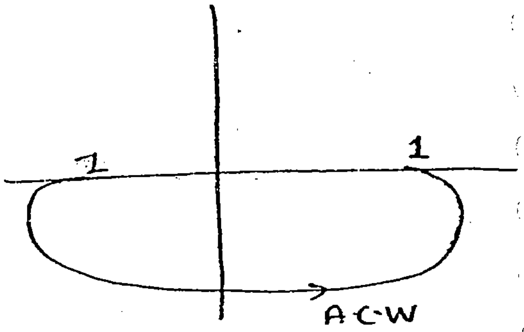
$$= -180^\circ + 2\tan^{-1}\omega/2$$

$\omega = 0$ $\quad 1\angle -180^\circ$

$\omega = \infty$ $\quad 1\angle 0^\circ$

E.D. → ACW

S.D → X



Q. $GH = \frac{(S+1)}{S(S-3)}$

$$\phi = -90^\circ - \left(180^\circ - \tan^{-1}\frac{\omega}{3}\right) + \tan^{-1}\omega$$

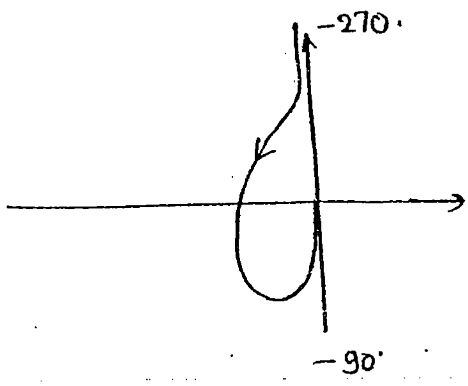
$$\phi = -270^\circ + \tan^{-1}\frac{\omega}{3} + \tan^{-1}\omega$$

$\omega = 0$ $\quad \infty\angle -270^\circ$

$\omega = \infty$ $\quad 0\angle -90^\circ$

E.D. → ACW.

S.D → Not required



Q. $GH = \frac{S+2}{S-2}$

NYQUIST PLOT —

Purpose →

* to draw the Complete freq. response of O.L.T.F.
* to find the no. of closed loop Poles in right Half S-plane
* to find the range of K-value for System Stability.
* to find the G.M., P.M., ω_{pc}, ω_{gc}.
* To find the Relative Stability by using G.M. and P.M.

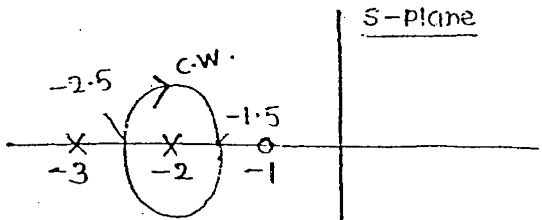
The Nyquist Stability Criteria is developed by using the mathematical principle known as principle of arguments.

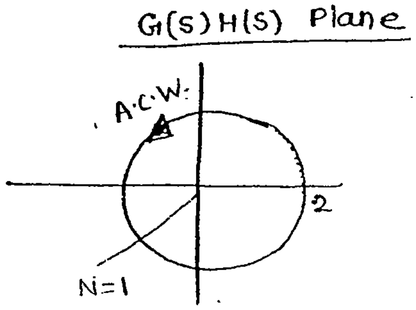
Principle of argument —

It states that If there are P poles and Z zero are enclos by the S-plane closed path then the Corresponding G(S) H(S) Plane encircles the origin with (P−Z) tin

$$\boxed{N = (P - Z)}$$

$$GH = \frac{(S+1)}{(S+2)\,(S+3)}$$

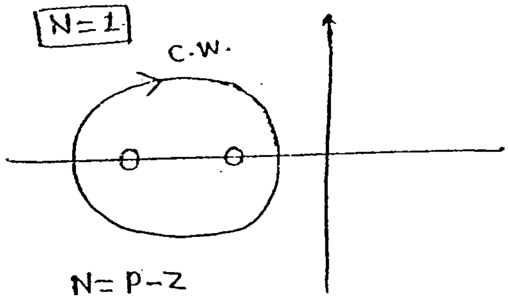


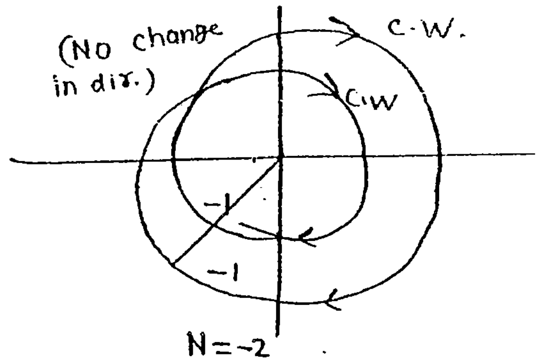


N = P − Z

N = 1 − 0

$\boxed{N = 1}$



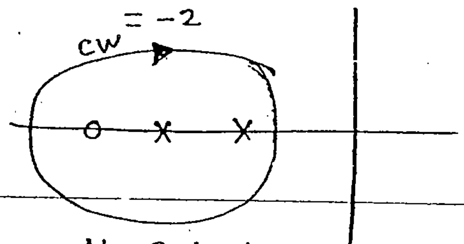


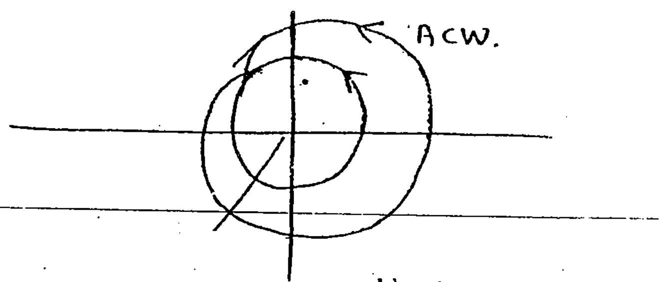
N = P − Z

N = 0 − 2

= −2

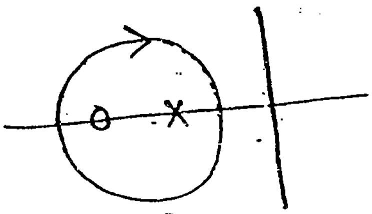
N = −2



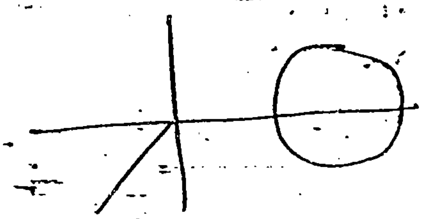


N = 3 − 1 = +2

N = +2



N=0

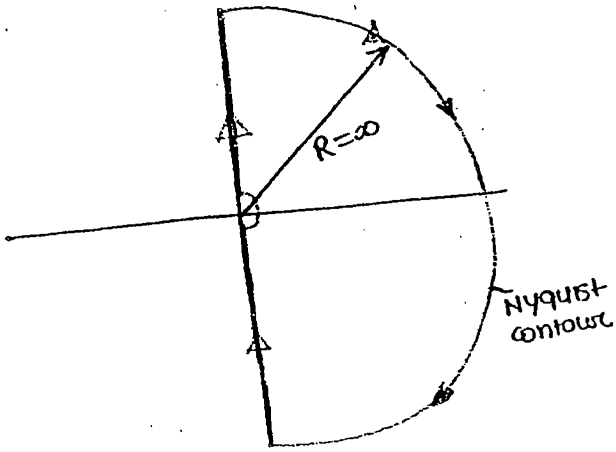


N=0.

P → change in direction

Z → no change in direction

ACW → +ve   CW → −ve



The principle of argum[ent] is applied to the total R-H of S-plane by sel[ecting] as a closed path.

The selected total Right [half] of S-plane as closed pa[th] called the Nyquist conto[ur]

Nyquist Stability criteria is Right Half S-plane analysis.

* Pole zero configuration :-

The OLTF . (Pole zero configuration)

$$G(S)H(S) = K\frac{N(S)}{D(S)} \quad \text{——— ①}$$

C.LTF

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{G(S)}{1+K\ldots}$$

$$= \frac{G(S)\,D(S)}{D(S)+KN\ldots}$$

the System Stability given by char. equ^n.

i.e.

$$q(S) = 1+G(S)H(S)$$

$$= 1+K\frac{N(S)}{D(S)} = \frac{D(S)+KN(S)}{D(S)} \quad \text{———}$$

compare ① and ③

* Poles of char. eq. = OLTF Poles.

compare ② and ③

char. eq. = CLTF Poles.

$N = P - Z$

P = Poles of char. eq. in Right Half of S-plane = OLTF Pole in R-H S plane

Z = zero of char. eq. in Right side of S-plane
= CLTF Pole in Right Half of S-plane

N = no. of encirclement about critical point $(-1+j0)$.

OL - Stability

O.L. poles = 0

Poles of C.E = 0.

P = 0

so

$N = P - Z$

$\boxed{N = -Z}$

CL - Stability

CL Pole = 0

zero of CE = 0

Z = 0

N = P.

* to become OL system stable there should not be any open loop pole in Right Half of S-plane.

* The OL pole is nothing but poles of C.E. which must be zero in Right side means P = 0 so

$\boxed{N = -Z}$

* to become CL System stable there should not be any closed loop pole in Right Half S-Plane, the closed loop pole is nothing but zero's of CE which must be zero in Right Half S-plane i.e.

Z = 0

so $\boxed{N = P}$

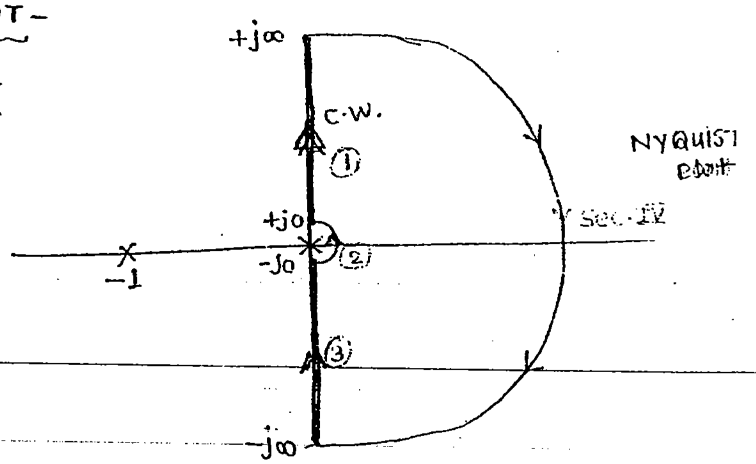
NYQUIST STABILITY CRITERIA -

The no. of encirclement about the $(-1+j0)$ must be equal to poles of char. equation which are nothing but OLTF poles in the Right Half S-plane i.e.

$\boxed{N = P}$

Draw the NYQUIST PLOT -

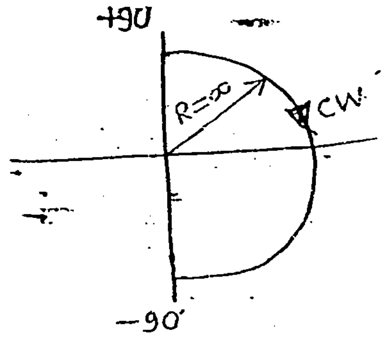
$$G(S)\,H(S) = \frac{1}{S(S+1)}.$$



Draw the Blar Plat

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

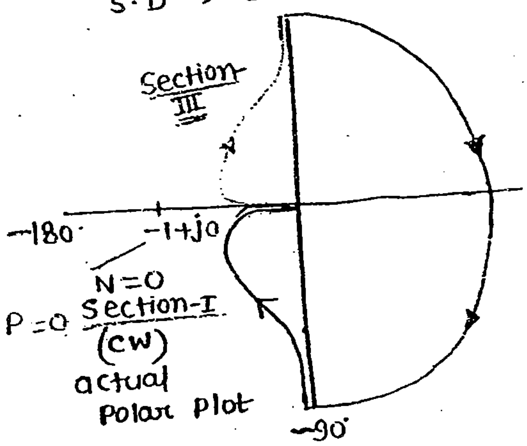
$$\phi = -90^\circ - \tan^{-1}\omega$$



Infinite radius of circle.

Section-I	
$\omega = 0$	$\infty \angle -90^\circ$
$\omega = \infty$	$0 \angle -180^\circ$

E.D. → C.W.
S.D → C.W.



$$\phi = \pm 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{\ldots}$$

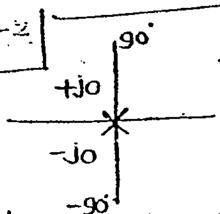
Because of Pole at origin

(The no. of '∞' Radius half circle (180°) circle = No. of Poles at origin)

{ The above statement is not valid for zero's, zero doesn't give ∞ Radius circle. Ex → $\frac{S}{S+1}$

$\omega = 0^+$ 0 } $\omega = \infty^+ \cdot 1$ }
$= 0^-$ 0 } $\omega = \infty^- \; 1$ }

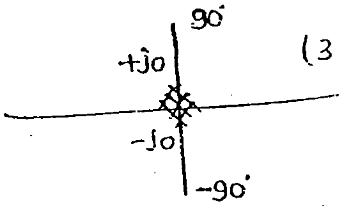
Section-2



$$\left.\frac{1}{S}\right|_{S=+j0} = \frac{1}{\angle +(j0)} = -\angle j0 = -90^\circ$$

$$\left.\frac{1}{S}\right|_{S=-j0} = \frac{1}{\angle -j0} = +\angle j0 = +90^\circ$$

180×2



(3 Poles at origin)

$$\frac{1}{S^3} = \frac{1}{3\angle +j0} = \frac{1}{3\times 90} = -270^\circ$$

$$= \frac{1}{3\angle -j0} = +270^\circ$$

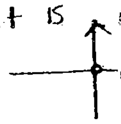
$$= 540^\circ$$

$$= 180\times 3$$

※ Section-3 is mirror image of Section 1, with respect to Real axis. But direction is continuous.

Section-4 →

The section-④ gives magnitude of zero Hence neglect the section ④ because it is point at origin.



Join Section ①, ②, ③

→

※ ※ The infinite Radius of circle should be start where mirror image is end and infinite radius of circle end where actual Blar plat started.

Infinite Radius circle direction always CW.

$\omega = 0^-$	$\infty \angle +90^\circ$
$\omega = 0^+$	$\infty \angle -90^\circ$

E.D. = C.W.

※ N = P (CL system stable)

CLTF $S^2+S+1=0$

$G(S)\,H(S) = \frac{\quad}{S+1}$

* Draw the polar plot

$$M = \frac{10}{\sqrt{\omega^2+1}}$$

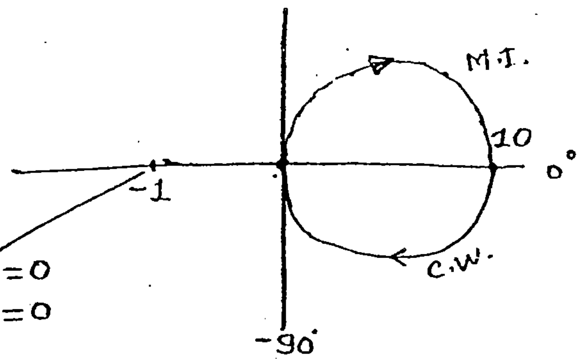
$$\angle\phi = -\tan^{-1}\omega$$

$\omega = 0$ $\quad 10\angle 0^\circ$

$\omega = \infty$ $\quad 0\angle -90^\circ$

ED = C.W.

S.D. = CW.



N=0

P=0

$\boxed{N=P}$ — CL-sys stable

Q.

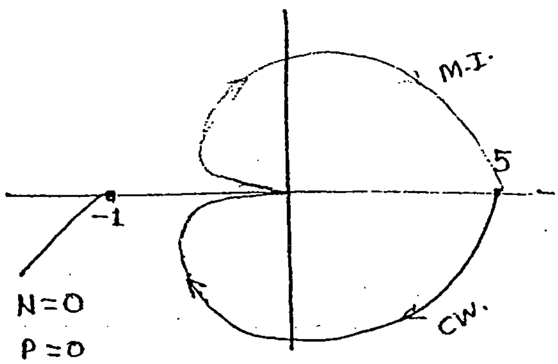
$$G(S)\,H(S) = \frac{10}{(S+1)\,(S+2)}$$

$$M = \frac{10}{\sqrt{(\omega^2+1)\,(\omega^2+4)}}$$

$$\angle\phi = -\tan^{-1}\omega - \tan^{-1}\omega/2$$

$\omega = 0$ $\quad 5\angle 0^\circ$ $\quad \left.\begin{matrix}\text{E.D.}\\ \text{S.D.}\end{matrix}\right\}$ CW.

$\omega = \infty$ $\quad 0\angle -180^\circ$



N=0

P=0

~~(N=P) (Stable)~~

Q.

$$G(S)\,H(S) = \frac{10}{S^2(S+2)\,(S+4)}$$

$$M = \frac{10}{\omega^2\sqrt{\omega^2+4}\,\sqrt{\omega^2+1}}$$

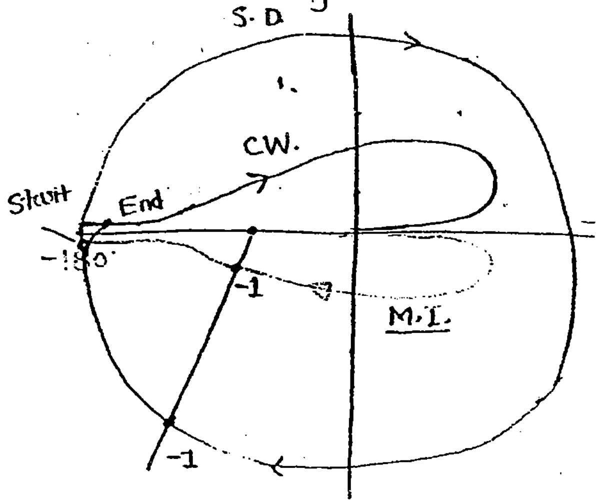
$$\angle\phi = -180 - \tan^{-1}\omega/2 - \tan^{-1}$$

at

$\omega = 0$ $\quad \infty\angle -180^\circ$

$\omega = \infty$ $\quad 0\angle -360^\circ$

$\left.\begin{matrix}\text{E.D.}\\ \text{S.D}\end{matrix}\right\}$ CW.



N = -2 , P = 0. closed loop system unst

N ≠ P

(there has two poles at origin)

the no. of closed loop poles in Right side given by Principle of Argument

$$N = P - Z$$

$$-2 = 0 - Z$$

$\boxed{Z = 2}$ closed loop Poles in Right Half S-Plane.

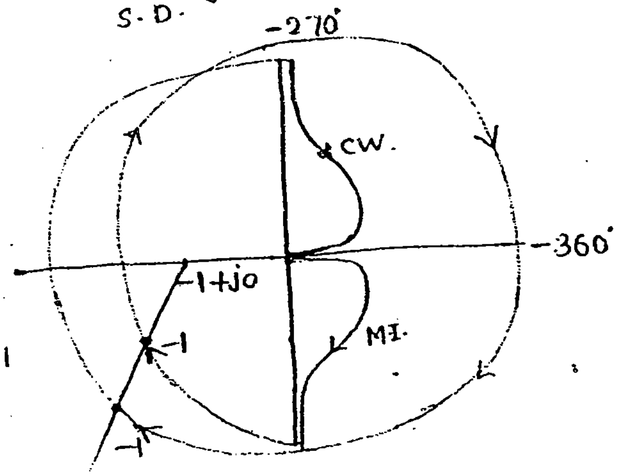
$$GH = \frac{1}{s^3(s+10)}$$

$$\angle\phi = -270^\circ - \tan^{-1}\left(\frac{\omega}{10}\right)$$

at $\omega = 0$ $\quad \infty \angle -270^\circ$

$\omega = \infty$ $\quad 0 \angle -360^\circ$

E.D. } CW.
S.D.



$N = -2$.

$P = 0$.

$N \neq P$ / CL System U.S.

$N = P - Z$

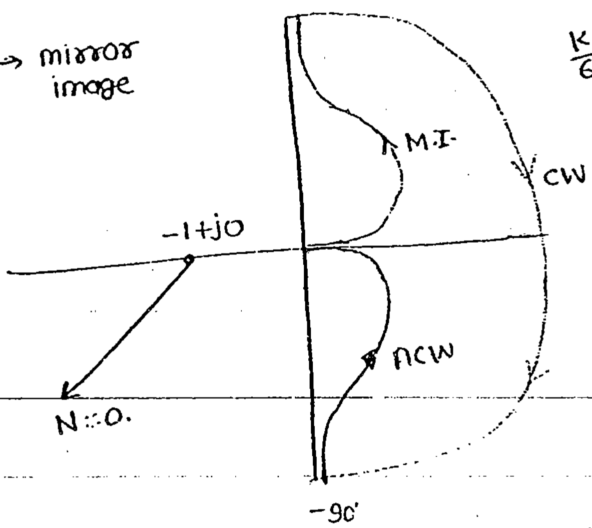
$-2 = 0 - Z$

$\boxed{Z = 2}$

Z=2 Closed loop Poles in Right Half S-Plane.

Q. $GH = \dfrac{1}{S(1-S)}$

M.I → mirror image



$$\angle\phi = -90^\circ - (-\tan^{-1}\omega).$$
$$= -90^\circ + \tan^{-1}\omega.$$

$\omega = 0$ $\quad \infty \angle -90^\circ$

$\omega = \infty$ $\quad 0 \angle 0^\circ$

ED ⇒ ACW

SO ⇒ X

$N = 0$

$P = 1$

$\boxed{N \neq P}$ Closed loop System unsta

Number of closed loop Pole at right side

$N = P - Z$

$0 = 1 - Z$

$\boxed{Z = 1}$

* Find the Range of K value for system Stability by using Nyquist Stability Analysis.

$$G(S)H(S) = \frac{K}{(S+1)(S+2)(S+3)}$$

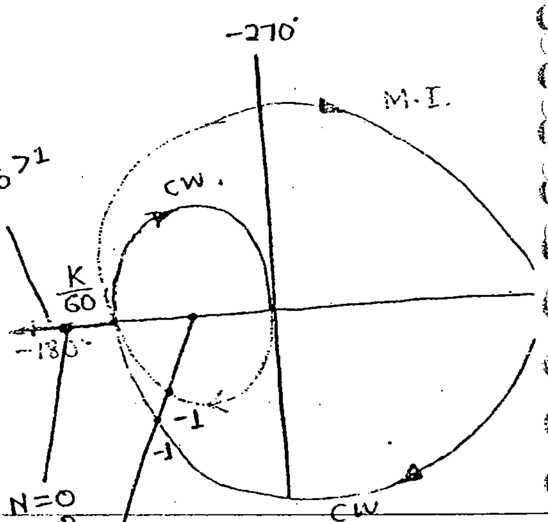
$$\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

$\omega = 0$ $\quad K/6 \angle 0^\circ$

$\omega = \infty$ $\quad 0 \angle -270^\circ$

E.D. → CW.

S.D → CW.



$N = 0$
$P = 0$
CL (S)

$\left.\begin{matrix} N = -2 \\ P = 0 \end{matrix}\right\}$ $N \neq P$ CL (U.S) $\quad \frac{K}{60} >$

$K > 6$

Assume that the intersection point with -180° equal to critical point that means, the magnitude of I.P. = magnitude of critical point = magnitude = 1

$$\left|\frac{K}{60}\right| = |-1+j0|$$

$$\frac{K}{60} = 1$$

* Shift the I.P. towards the $-\infty$ by considering magnitude >1 in this case the critical point is inside the loop, for this get the no. of encirclements and condition for the stability.

* Shift the intersection point towards the zero by considering magnitude is $<$ than 1. in this case the critical point is outside the loop for this get the no. of encirclement and condition for stability.

$$\left.\begin{array}{l} N=0 \\ P=0 \end{array}\right\} N=P.$$

Closed loop system stable

$$\frac{K}{60} < 1$$

$$\boxed{K < 60}$$

***

whenever the stability condition is less than the certain value then the other limit is decided by the intersection point with 0°. the intersection point with 0° must be always greater than -1 in the above problem.

$$\frac{K}{6} > -1$$

$$K > -6$$

$$\boxed{-6 < K < 60}$$

(Stable)

Find the range of K value for

$$G(s)H(s) = \frac{K(s+3)}{s(s-1)}.$$

$$\angle\phi = -90^\circ - (180^\circ - \tan^{-1}\omega) + \tan^{-1}\frac{\omega}{3}$$

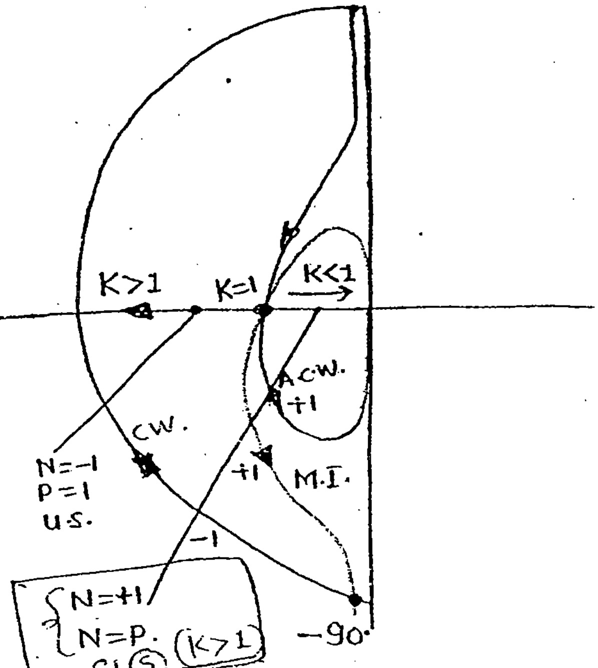
$$= -270^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}.$$

$\omega = 0$ $\quad \infty \angle -270^\circ$

$\omega = \infty$ $\quad 0 \angle -90^\circ$

E.D $\rightarrow$ ACW

S.D. $\rightarrow$ Not required.



$$\angle GH = -180^\circ$$

$$-180^\circ = -270^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3}$$

$$90^\circ = \tan^{-1}\frac{\omega + \omega/3}{1 - \omega^2/3}$$

$$\boxed{\omega = \sqrt{3} \text{ rad/sec}}$$

$$M = \frac{K\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}}$$

$$M\Big|_{\omega=\sqrt{3}} = K$$

$\boxed{K>1} \rightarrow$ CL system stable.

$K < 1$

$N = P - Z$

$-1 = 1 - Z$

$\boxed{Z = 2}$ 2 CLP R.H.S. of S-plane.

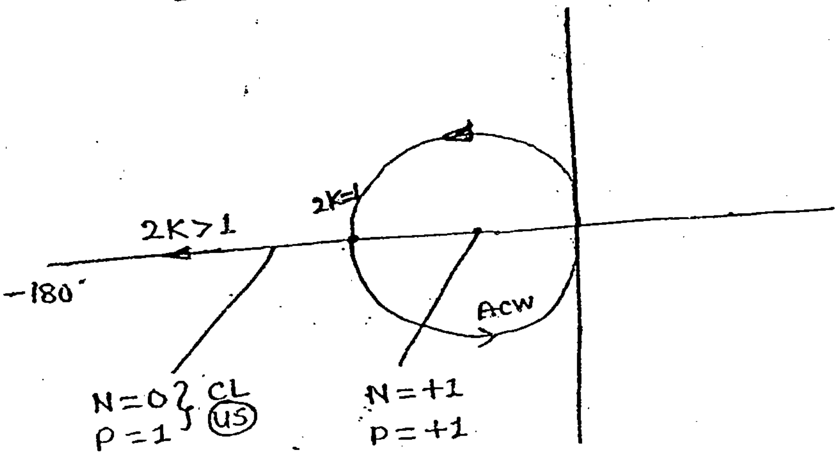
Q. $G(s)\,H(s) = \dfrac{K(s+2)}{(s+1)(s-1)}$

$\angle\phi = -\tan^{-1}\omega - (180 - \tan^{-1}\frac{\omega}{1}) + \tan^{-1}\omega/1$

$= 1\tan^{-1}\frac{\omega}{1} - \tan^{-1}\omega - 180^\circ + \tan^{-1}\omega.$

$\omega = 0$ $\quad +2K \angle -180^\circ$

$\omega = \infty$ $\quad 0 \angle -90^\circ$



$N = 0, P = 1$ } CL (US)

$2K < 1$

$\boxed{K < 1/2}$

$N = +1$, $P = +1$

$\boxed{N = P}$

CL (S)

$2K > 1$

$\boxed{K > 1/2}$

Q. $G(s)\,H(s) = \dfrac{K(s-2)}{s+2}$

$M = \dfrac{K\sqrt{\omega^2+4}}{\sqrt{\omega^2+4}}$

$\angle\phi = -\tan^{-1}\frac{\omega}{2} + 180^\circ - \tan^{-1}\frac{\omega}{2}$

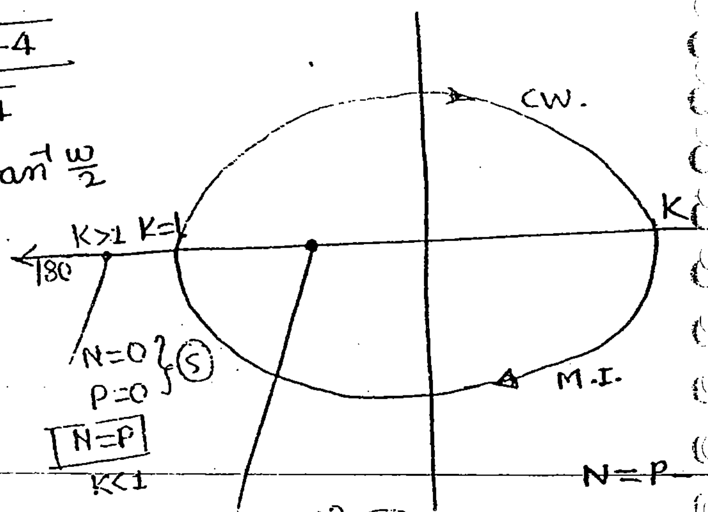
$= 180^\circ - 2\tan^{-1}\frac{\omega}{2}$

$\omega = 0$ $\quad K\angle 180^\circ$

$\omega = \infty$ $\quad K\angle 0^\circ$

E.D. = CW.

S.D := Not req.



$N = 0, P = 0$ } (S)

$\boxed{N = P}$

$K < 1$

$N = -1, P = 0$ } (U.S)

($K > 1$)

$N = P -$

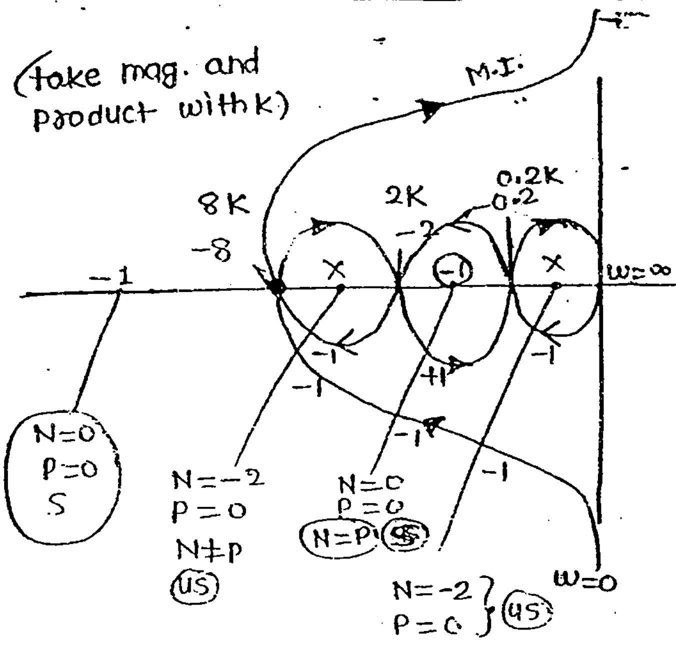
$-1 = 0 -$

$\boxed{Z = 1}$ C

$-1 < K < 1$ (5)

→ The OLTF to the given.
* The polar plot shown in figure is stable then the range of K value for CL system stability is.

(take mag. and product with K)



x → invalid critical point

1 is existing in this Region when

$0.2K < 1$ and $2K > 1$

$\boxed{K < 5}$ & $\boxed{K > 0.5}$

→ $\boxed{0.5 < K < 5}$

→ $8K < 1 \Rightarrow \boxed{K < 1/8}$

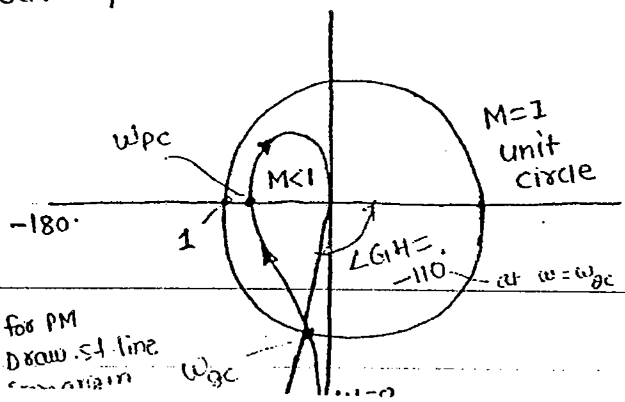
So Ans.

$\boxed{K < 1/8}$ and $\boxed{0.5 < K < 5}$

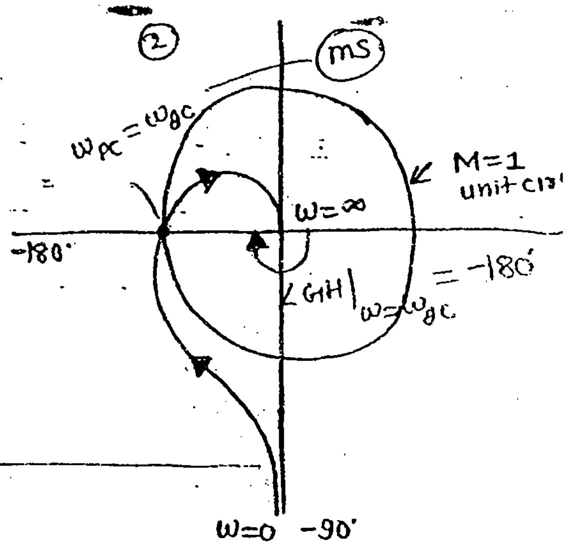
Stability

Identify the stability to the given plots.

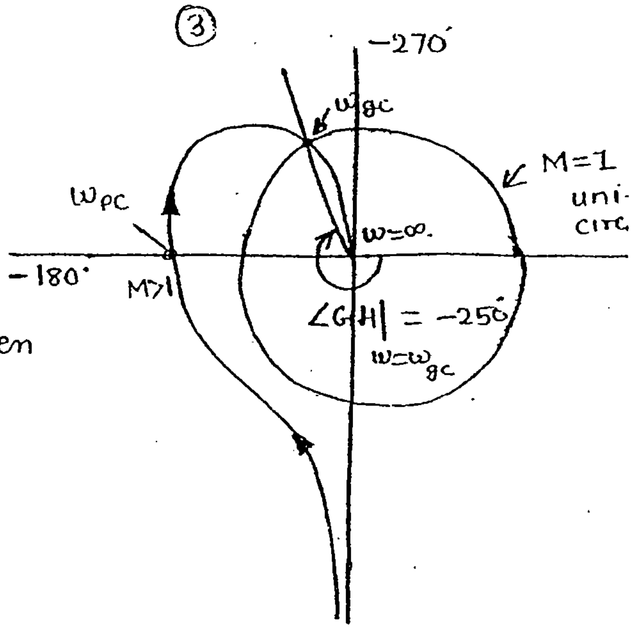
(1)



(2)



(3)



(1) $\omega_{pc} > \omega_{gc}$ System –

$$GM = \frac{1}{M|_{\omega_{pc}}} = \frac{1}{<1} = >$$

$$PM = 180° + \angle GH|_{\omega_{gc}}$$

$$= 180° - 110°$$

$$= 70°$$

(2) $GM = \frac{1}{1} = 1$

$PM = 180 - 180° = 0$ } (M

(3) $GM = \frac{1}{>1} = <1$

$PM = 180° - 250° = -70°$

(US)

axis with magnitude ...
then system is Stable, because in this case $\boxed{\omega_{pc} > \omega_{gc}}$

* whenever the plot intersect −180 axis with magnitude = 1 then system is m.s. because $\boxed{\omega_{pc} = \omega_{gc}}$

* whenever plot intersect −180° line with magnitude > 1 then the system is unstable because $\boxed{\omega_{pc} < \omega_{gc}}$

* Calculations of G.M. and P.M.

① calculate the G.M. for

$$G(S)H(S) = \frac{1}{S(S+1)(S+2)}$$

$$GM = \frac{1}{|G(S)H(S)|}\Bigg|_{\omega=\omega_{pc}} = -20\log|GH(j\omega)|_{\omega=\omega_{pc}}$$

(S1) find $\omega_{pc} \Rightarrow \angle GH = -180°$

(S2) $|M|_{\omega_{pc}}$

(S3) GM.

$\omega_{pc} \rightarrow$

$$\angle GH = -180°$$

$$180° = 90 + \tan^{-1}\omega + \tan^{-1}\omega/2$$

$$90° = \tan^{-1}\frac{\omega+\omega/2}{1-\omega^2/2}$$

$$\boxed{\omega = \omega_{pc} = \sqrt{2}\ \text{rad/sec.}}$$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$M\Big|_{\omega=\omega_{pc}} = \frac{1}{\sqrt{2}\sqrt{3}\sqrt{6}} = \frac{1}{6}$$

$$GM = \frac{1}{M|_{\omega=\omega_{pc}}} = 6$$

$$= -20\log 1/6$$

$$= 15.56\ dB.$$

100 ...

$$G(S)H(S) = \frac{1}{S(S+1)}$$

$$PM = 180° + \angle GH(j\omega)\Big|_{\omega}$$

(S1) find ω_{gc} $|M| = 1$

(S2) PM

$$\frac{1}{\omega\sqrt{\omega^2+1}} = 1$$

$$\omega^2(\omega^2+1) = 1$$

$$\omega^4 + \omega^2 - 1 = 0$$

$$x = \omega^2$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$\omega^2 = \frac{-1 \pm \sqrt{5}}{2}$$

$$\omega_{gc} = 0.786\ \text{rad/sec.}$$

$$PM = 180° - 90 - \tan^{-1}\cancel{\omega}\ 0.7$$

$$= \underline{52°}$$

③ Find the k value to get PM = 45° is.

$$G(S)H(S) = \frac{K}{S(S+1)}$$

$$45° = [-90° - \tan^{-1}\omega] + 18$$

$$\tan^{-1}\omega = 45$$

$$\omega = \omega_{gc} = 1\ \text{rad/sec.}$$

$$\frac{K}{\omega\sqrt{\omega^2+1}} = 1$$

$$\boxed{K = \sqrt{2}}$$

(4) Find the K value to get GM = 20dB.

$$G(S)H(S) = \frac{K}{S(S+2)(S+4)}$$

ⓐ

$$\angle G(S)H(S) = -90° - \tan^{-1}\omega/2 - \tan^{-1}\omega/4$$

$$60° = 180° - 90° - \tan^{-1}\omega/2 - \tan^{-1}\omega/4$$

$$\tan 30 = \frac{\omega/2 + \omega/4}{1 - \omega^2/8}$$

$$\frac{1}{\sqrt{3}} = \frac{6\omega}{\cancel{8}} \times \frac{\cancel{8}}{8-\omega^2}$$

$$8 - \omega^2 = 6\sqrt{3}\,\omega$$

$$\omega^2 + 6\sqrt{3}\,\omega - 8 = 0.$$

$$\boxed{\omega = 0.719 \text{ rad/sec}}$$

$$1 = \frac{K}{\omega\sqrt{(\omega^2+4)(\omega^2+16)}}\Bigg|_{\omega=0.72}$$

$$\boxed{K = 62}$$

ⓑ

$$-180° = -90° - \tan^{-1}\omega/2 - \tan^{-1}\omega/4$$

$$\frac{1}{0} = \frac{\omega/2 + \omega/4}{1 - \omega^2/8}$$

$$\boxed{\omega = 2\sqrt{2} \text{ rad/sec.}}$$

$$-20\log\frac{K}{55.42} = 20$$

$$0.1 = \frac{K}{\sqrt{8}\sqrt{12\times24}}$$

$$\boxed{K = 4.8}$$

(*) the O.L.T.F. of u.f.b. control system is given by

$$G\,H(S) = \frac{aS+1}{S^2}$$

the value of a to get PM = 45°

$$\angle GH(S) = -180° + \tan^{-1}(a\omega)$$

$$45° = \cancel{180} - \cancel{180} + \tan^{-1}a\omega$$

$$1 = +a\omega \qquad \boxed{\omega = 1/...}$$

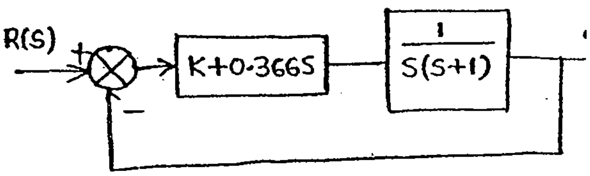
magnitude condition.

$$\left.\frac{\sqrt{(a\omega)^2+1}}{\omega^2}\right|_{\omega=1/a} = 1$$

$$\frac{\sqrt{2}}{(1/a)^2} = 1$$

$$\boxed{a = \sqrt{1/\sqrt{2}} = 0.8}$$

(*) If the compensator system shown in figure has a PM= at a crossover freq. of 1 r the value of system gain K



$$G(S)H(S) = \frac{(K+0.366S)}{S(S+1)}$$

$$\angle GH(S) = -90° - \tan^{-1}\omega + \tan^{-1}\left(\frac{0.}{}\right.$$

to calculate G.M., P.M. we requ OLTF, but the stability condit is for closed loop.

$$60° = 180° - 90° - \tan^{-1}\omega + \tan^{-1}\frac{0.36}{K}$$

$$-30° = \tan^{-1}\frac{\frac{0.366\omega}{K} - \omega}{1 + \frac{0.366\omega^2}{K}}$$

$$\frac{1}{\sqrt{3}} = \left.\frac{\omega - \frac{0.366\omega}{K}}{1 + \frac{0.366\omega^2}{K}}\right|_{\omega=}$$

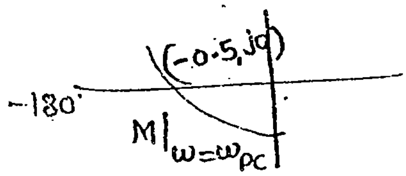
$$\frac{1}{\sqrt{3}} = \frac{1 - \frac{0.366}{K}}{1 + \frac{0.366}{K}}$$

$$\boxed{K = 1.366}$$

* in GH plane the Nyquist plot of Loop T.F.

$$GH = \frac{\pi e^{-0.25S}}{S}$$

* passes through −ve Real axis at the point is.



* It is an intersection point with −180° $\omega_{pc} \rightarrow$

convert rad into degree

$\angle GH = -180^\circ$

$$-180^\circ = -90 - 0.25\omega \times \left(\frac{180}{\pi}\right)$$

$$90 = \frac{45\omega}{\pi}$$

$\boxed{\omega_{pc} = 2\pi}$ rad/sec.

mag.

$M = \pi/\omega_{pc}$

$M = \frac{\pi}{2\pi}$

$\boxed{M = 1/2}$

I.P. $\rightarrow (-0.5, j0)$

* calculate GM and PM to the above problem.

$$GM = \frac{1}{M|_{\omega_{pc}}} = 2.$$

for PM ⇒

$\omega_{gc} \Rightarrow$

$M = 1$

$\pi/\omega = 1$

$\boxed{\omega_{gc} = \pi \text{ rad/sec.}}$

$$PM = 180^\circ - 90^\circ - 0.25\,\omega \times \frac{180}{\pi}$$

$= 45^\circ$

calculate GM and ...

$$GH(S) = \frac{e^{-S}}{S(S+1)}$$

$\boxed{GM}$

$\angle GH = -180^\circ$

$$-180^\circ = -90^\circ - \tan^{-1}\omega - \omega$$

$$90^\circ = \tan^{-1}\omega + \frac{180\omega}{\pi}$$

$\boxed{\omega_{pc} = 0.86 \text{ rad/sec.}}$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}\Bigg|_{\omega_{pc} = 0.86}$$

$= 0.88$

$\boxed{GM = 1.13}$

$\boxed{PM}$

$\omega_{gc} \rightarrow$

$M = 1$

$$\frac{1}{\omega\sqrt{\omega^2+1}} = 1$$

$\boxed{\omega_{gc} = 0.786 \text{ rad/}}$

$$PM = 180^\circ - 90^\circ - \tan^{-1}\omega - \omega \times$$

$\boxed{PM = 6.78}$

* Draw the polar plot

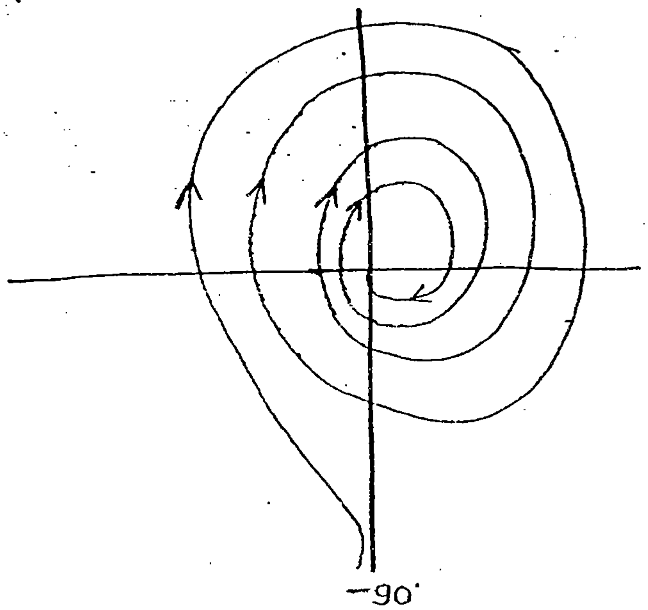
$$G(S)H(S) = \frac{\pi e^{-0.25S}}{S}$$

$M = \pi/\omega$

$$\angle\phi = -90^\circ - 0.25\,\omega \times \frac{18}{\pi}$$

$$= -\frac{45^\circ\omega}{\pi} - 90^\circ$$

ω	M	$\angle$
0	∞	−
1	π	−
10	0.1π	−
100	0.01π	

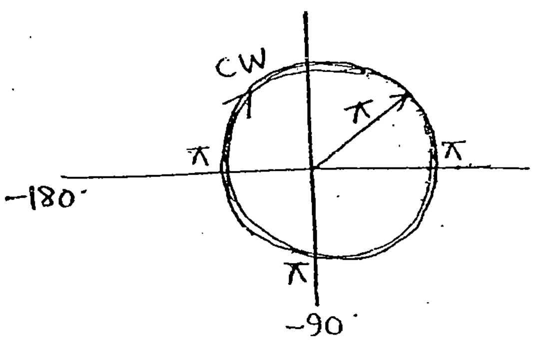


(*) $GH = \pi e^{-0.25S}$

$M = \pi$

$\angle\phi = -0.25\omega \times \frac{180°}{\pi}$

$= \frac{-45°\omega}{\pi}$ (Negative angle) CW



(*) calculate GM. and P.M.

$GH(S) = \frac{1}{S+2}$

$\boxed{GM}$ $M = \frac{1}{\sqrt{\omega^2+4}}$

$\omega_{pc} \rightarrow \angle GH = -180°$

$\boxed{-\tan^{-1}\omega/2 \cancel{=} -180°}$

* $\omega = 0 \Rightarrow 0°$
* $\omega = \infty \Rightarrow -90° < -180°$ so $\omega_{pc} = \infty$

GM

$M|_{\omega_{pc}} = 0$

so

$\boxed{GM = \infty}$

$\boxed{PM} \Rightarrow$

$\omega_{gc} \Rightarrow$

$M = 1$

$\frac{1}{\sqrt{\omega^2+4}} = 1$

$\omega^2 + 4 = 0$

$\boxed{\omega = \pm 2i}$ X

$\omega \rightarrow$ Never be Imagenary

$\omega = 0$	$M = 0.5$
$\omega = \infty$	0

* $M < 1$
* $\omega_{gc} = 0$
* $PM = \infty$.

* whenever plot or T.F. gives less magnitude than 1 then the $\omega_{gc} = 0$. and PM become ∞.

Q. $GH = 1/S,\ 1/S^2,\ 1/S^3$

$\boxed{GM} \Rightarrow$

$\omega_{pc} \rightarrow$

$\angle GH = -180°$

$-90° = -180°$ X

$< -180° \longrightarrow \omega_{pc} = \infty$

$M = \frac{1}{\omega_{pc}} = 0$

$\boxed{GM = \infty}$

$\boxed{PM} \Rightarrow$

$\omega_{gc} \longrightarrow M = 1$

$1/\omega = 1$

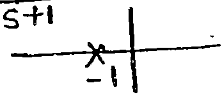
$\omega_{gc} = 1$ rad/sec

$PM = 180^\circ - 90^\circ$

$\boxed{PM = 90^\circ}$

Stable

$CL = \frac{1}{s+1}$



② $GH = 1/s^2$

GM $\rightarrow$

$\omega_{pc} \Rightarrow$

$\angle GH = -180^\circ$

$-180^\circ = -180^\circ$

$\boxed{\omega_{pc} = \omega_{gc}}$ m.s.

$\omega_{gc} \Rightarrow$

$M = 1$

$\frac{1}{\omega^2} = 1$

$\boxed{\omega_{gc} = \omega_{pc} = 1}$

$\boxed{GM = 1}$

$PM = 180^\circ - 180^\circ$

$\boxed{PM = 0}$

$\Rightarrow \boxed{m.s.}$

③ $GH = 1/s^3$

GM $\rightarrow$

ω_{pc}

$\angle GH = -180^\circ$

$-270^\circ \neq -180^\circ$

$> -180^\circ \xrightarrow{} \omega_{pc} = 0$

$M = \infty$

$GM = 0 \; (<1) \longrightarrow$ U.S.

PM

$M = 1$

$\boxed{\omega_{gc} = 1}$

$PM = 180^\circ - 270^\circ$

$= -90^\circ$

$=$ U.S.

State - Space Analysis

State gives the future Behaviour of the System Based on Present i/p and past History of the system.

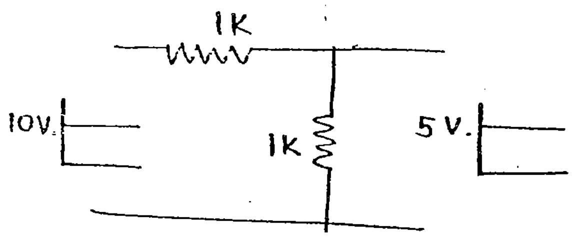
$\boxed{\text{State}}$

↓

Future Behaviour

↓

- Past i/p
- Past History of System. (Initial State (cond.))
  - (State variable) $V_C \rightarrow$ cap. $I_L \rightarrow$ Ind.



$V_o = 0.5V_i$

The past History of the system is described by State variables the memoryless sys. (Resistive n/w) are not having any State variables because no energy stored in the Res. n/w.

No. of State variables —

* If electrical n/w is given, the no. of State variable = Sum of inductors and capacitors.
* If the D.E. is given the no. of State variables = order of the D
* The Standard form of State model is

$$\dot{x} = Ax + Bu \quad \text{— state eq}^n / \text{Dynamic equation.}$$
$$y = Cx + Du. \quad \text{— o/p eq}^n$$

$x \rightarrow$ State vector $\quad u \rightarrow$ i/p vector

$\dot{x} \rightarrow$ Differential State vector.

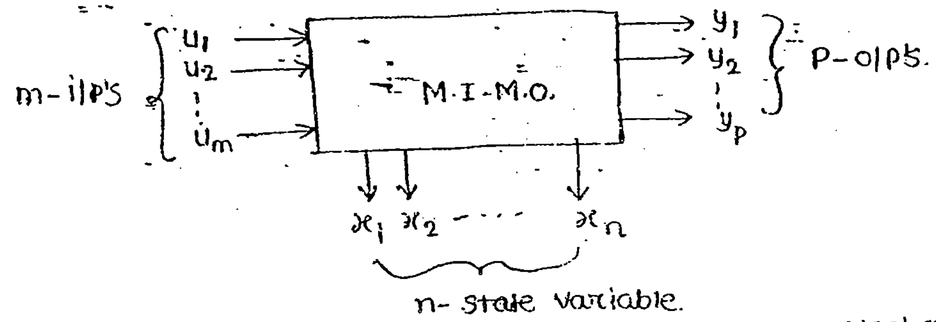
$y \rightarrow$ o/p vector $\quad D \rightarrow$ Transmission matrix.

$C \rightarrow$ o/p matrix

Note — The Tx. matrix is always zero If the ckt. not consist any active element.

* **order of matrix →**

consider the Multi i/p multi o/p system.



* **Input vector**

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}_{m\times 1} \qquad S.V.\ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$

o/p vector

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$$

$$\underbrace{\dot{X}}_{n\times 1} = \underbrace{A}_{n\times n}\underbrace{X}_{n\times 1} + \underbrace{B}_{n\times m}\underbrace{U}_{m\times 1}.$$

The order of differential state vector must be equal to order of state vector.

$$\underbrace{Y}_{p\times 1} = \underbrace{C}_{p\times n}\underbrace{X}_{n\times 1} + \underbrace{D}_{p\times m}\underbrace{U}_{m\times 1}.$$

* Find the order of matrices to the Following System.

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5 = 10u.$$

$n = 3 \qquad u = 1, \qquad y = 1$

$$\underbrace{\dot{X}}_{3\times 1} = \underbrace{A}_{3\times 3}\underbrace{X}_{3\times 1} + \underbrace{B}_{3\times 1}\underbrace{U}_{1\times 1}. \qquad \underbrace{Y}_{1\times 1} = \underbrace{C}_{1\times 3}\underbrace{X}_{3\times 1} + \underbrace{D}_{1\times 1}\underbrace{U}_{1\times 1}$$

State model to the D.E^n

* obtain the State model to the following System

$$\dddot{y} + 5\ddot{y} + 3\dot{y} + 10y = 5u.$$

with opposite sign.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{n = 3}$

Let $y = x_1$ ——— (1)

$$x_2 = \dot{y} = \dot{x}_1 \quad \text{——②} \quad \text{three state variable} = x_1\ x_2\ x_3$$

$$x_3 = \ddot{y} = \dot{x}_2 \quad \text{——③} \quad \text{three differential SV} = \dot{x}_1, \dot{x}_2, \dot{x}_3$$

$$\dddot{y} = \dot{x}_3 \quad \text{——④}$$

*** to get the $\dot{x}_3$ in terms of state variable substitute all the 4 equation in the given system.

$$\dot{x}_3 + 5x_3 + 3x_2 + 10x_1 = 5u.$$

$$\dot{x}_3 = 5u - 10x_1 - 3x_2 - 5x_3 \quad \text{——⑤}$$

* State model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} [u]$$

i/p coeffic

$$[y] = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\text{always.}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

* $\ddddot{y} + 2\dddot{y} - 4\ddot{y} + 6y = 10u.$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6 & 4 & +4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} [u]$$

controllable canonical form

$$[y] = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$A^T = \text{observable canonical form} = \begin{bmatrix} 0 & 0 & 0 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & +4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

* State model to the T.F. →

→ write the state model to the following system.

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}$$

$$\dddot{Y} = \dot{X}_3 = s^3$$

** $\boxed{s^n = \dot{X}_n}$

no. of state variable $= 2$
so we write $\dot{X}_2$ $\boxed{\text{but not } X_3}$.

$$\frac{Y(s)}{U(s)} = \frac{2s + 3}{s^2 + 5s + 6}$$

$\dot{x}_1 = x_2$, x_1

$\dot{x}_2$, $\dot{x}_1 = x_2$, x_1

$Y = 2X_2 + 3X_1$ ——— ①

$u = \dot{X}_2 + 5X_2 + 6X_1$

$\dot{X}_2 = u - 5X_2 - 6X_1$ ——— ②

State model

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1(K) \end{bmatrix} [u]$$

$$[Y] = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

→ Deno. of T.F. gives A matrix
Related to loops.
A matrix completely
(Lowest to Highest with opposite sign)

→ Numerator gives C matrix
(with same sign)

$$\frac{Y(s)}{U(s)} = \frac{K(2s+3)}{s^2+5s+6}$$

Q.

$$\frac{Y(s)}{U(s)} = \frac{10(2s^3+4s^2+6)}{s^4+3s^3+6s^2+9}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & 0 & -6 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \qquad c = \begin{bmatrix} 6 & 0 & 4 & 2 \end{bmatrix}$$

* $$\frac{Y(S)}{U(S)} = \frac{1}{(S+1)\ (S+2)\ (S+3).}$$ (Diagonalization form)

$$\frac{Y(S)}{U(S)} = \frac{(1/2)}{S+1} + \frac{(-1)}{S+2} + \frac{(1/2)}{S+3}$$

$$Y = \frac{(1/2)\,u}{S+1} + \frac{(-1)\,u}{S+2} + \frac{(1/2)\,u}{S+3}$$

$$Y = x_1 + x_2 + x_3.$$

$x_1 = \frac{(1/2)\,u}{S+1}$	$x_2 = \frac{(-1)\,u}{S+2}$	$x_3 = \frac{(1/2)\,u}{S+3}$
$S x_1 + x_1 = (1/2)\,u$	$S x_2 + 2x_2 = (-1)u.$	$S x_3 + 3x_3 = (1/2)u$
$\dot{x}_1 + x_1 = \left(\frac{1}{2}\right)u.$	$\dot{x}_2 + 2x_2 = (-1)u$	$\dot{x}_3 + 3x_3 = (1/2)u.$

State model -

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/2 \\ -1 \\ 1/2 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→* B and c matrices are interchangable

** Jordon canonical Form →

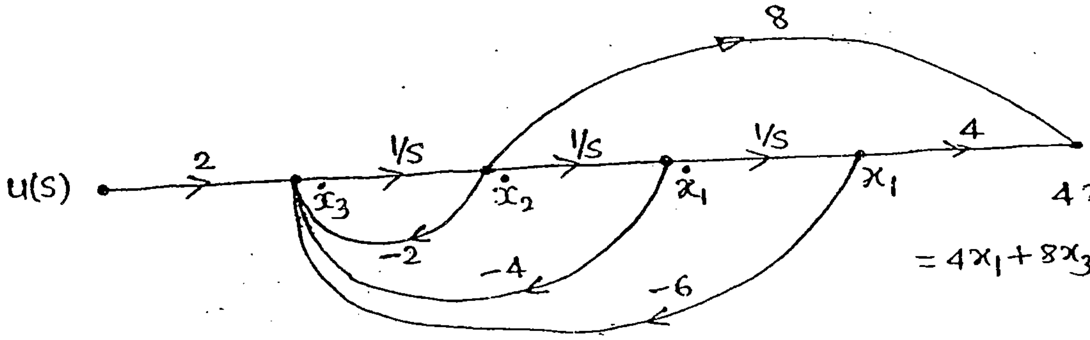
$$\frac{Y(S)}{U(S)} = \frac{1}{(S+2)^3\ (S+4)}$$

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

whenever same no. Repeated above that put +1 and remain all 0.

** State model to SFG →

→ write the state model to the following SFG.



$4x_1 + 8$

$= 4x_1 + 8x_3$ (Start all from S

(always consider incoming Branch

$$\dot{x}_1 = \frac{1}{s} \cdot \dot{x}_2 = x_2$$

$$\dot{x}_2 = \frac{1}{s} \dot{x}_3 = x_2$$

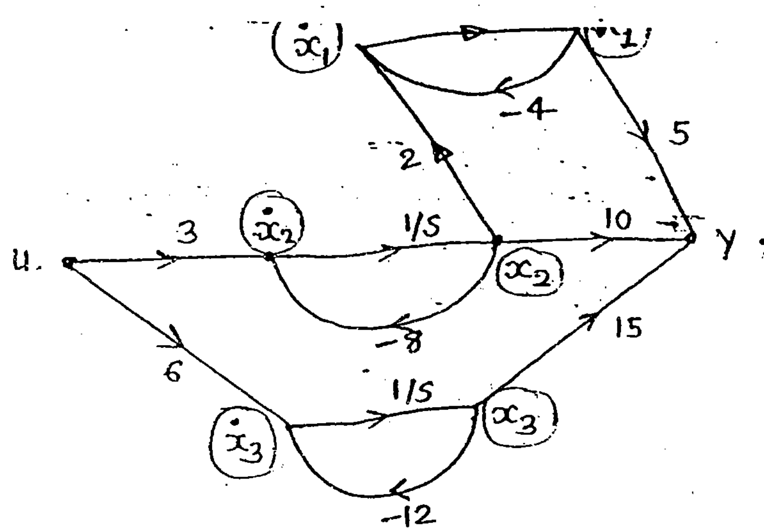
$$\dot{x}_3 = 2u - 2\dot{x}_2 - 4\dot{x}_1 - 6x_1$$

$$\dot{x}_3 = 2u - 2x_3 - 4x_2 - 6x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 4 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q.



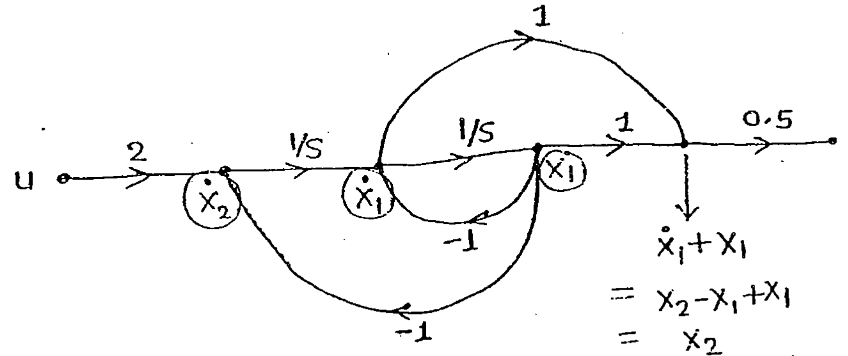
$Y = 5X_1 + 10X_2 \cdot 15X_3$.

$$\dot{x}_1 = 2x_2 - 4x_1$$
$$\dot{x}_2 = 3u - 8x_2$$
$$\dot{x}_3 = 6u - 12X_3.$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 5 & 10 & 15 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Q. SFG shown in figure, its state model is.



$Y = 0.5X_2$

$\dot{X}_1 + X_1$
$= X_2 - X_1 + X_1$
$= X_2$

$$\dot{X}_1 = X_2 - X_1$$
$$\dot{X}_2 = 2u - X_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

(Not Imp.) *

** State model to the electrical N/W →

Procedure →

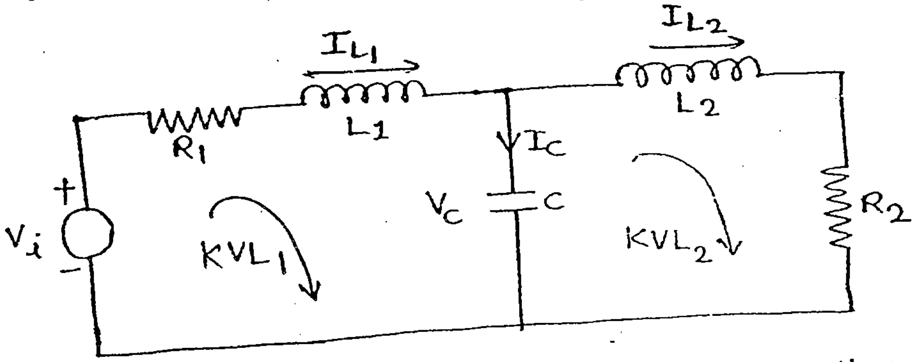
① Select the state variables as voltage across cap. or current through Inductor.

② no. of State variables = sum of Inductors and capacitors.

③ Apply KVL and KCL. at capacitor junction apply KCL and KVL through inductor.

④ The Resultant equation should consist state variable, Diff. state var., i/p var. and o/p. variable.

* write the state model to the given electrical model :



S.V. → $\begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix}$

→ KCL at capacitor Junction

$$I_c = I_{L_1} - I_{L_2}$$

$$C \frac{dV_c}{dt} = I_{L_1} - I_{L_2}$$

$$\dot{V}_c = \frac{I_{L_1}}{C} - \frac{I_{L_2}}{C} \qquad \text{—— ①}$$

KVL 1

$$V_i = R_1 I_{L_1} + L_1 \frac{dI_{L_1}}{dt} + V_c$$

$$\dot{I}_{L_1} = \frac{V_i}{L_1} - \frac{R_1}{L_1} I_{L_1} - \frac{V_c}{L_1} \qquad \text{—— ②}$$

KVL 2

$$V_c = L_2 \frac{dI_{L_2}}{dt} + R_2 I_{L_2}$$

$$\dot{I}_{L_2} = \frac{V_c}{L_2} - \frac{R_2}{L_2} \qquad \text{——}$$

state model →

$$\begin{bmatrix} \dot{V}_c \\ \dot{I}_{L_1} \\ \dot{} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ & & -R_2 \end{bmatrix} \begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ V_L \\ 0 \end{bmatrix}$$

$$[V_o] = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ I_{L_1} \\ I_{L_2} \end{bmatrix}$$

* <u>Transfer Function to the State model</u>

$$T.F. = C\,[SI - A]^{-1} B + D.$$

$$\boxed{T.F. = C \,\frac{\operatorname{adj}[SI-A]}{|SI-A|}\, B + D.}$$

$$|SI - A| = 0 \longrightarrow C.E.$$

Roots of C.E. is nothing But closed loop poles which are called eigen value.

→ Find the T.F. to the given State model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$[SI - A] = \begin{bmatrix} S+2 & +3 \\ -4 & S-2 \end{bmatrix}$$

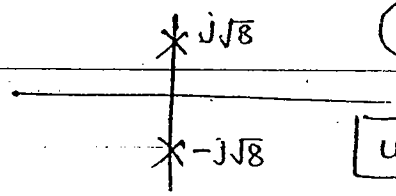
to get $(SI-A)$, add S diagonally and change the Sign of coefficients.

$$T.F. = \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} S-2 & -3 \\ +4 & S+2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{S^2 - 4 + 12}$$

$$= \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 3S-6-15 \\ 12+5S+10 \end{bmatrix}}{S^2+8} = \frac{8S+1}{S^2+8}.$$

<u>C.E.</u> $S^2 + 8 = 0$

$$\boxed{S = \pm j\sqrt{8}}$$



m.s.

undampe

* Find the

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 3\\ -2 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}[u]$$

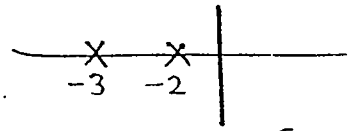
$$[y] = \begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\Rightarrow \frac{\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}s+5 & +3\\ -2 & s\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}}{s^2+5s+6} = \frac{\begin{bmatrix}2 & 1\end{bmatrix}\begin{bmatrix}s+8\\ s-2\end{bmatrix}}{(s+2)(s+3)}$$

$$\Rightarrow \frac{3s+14}{(s+2)(s+3)}$$

C.E. $(s+2)(s+3)=0$

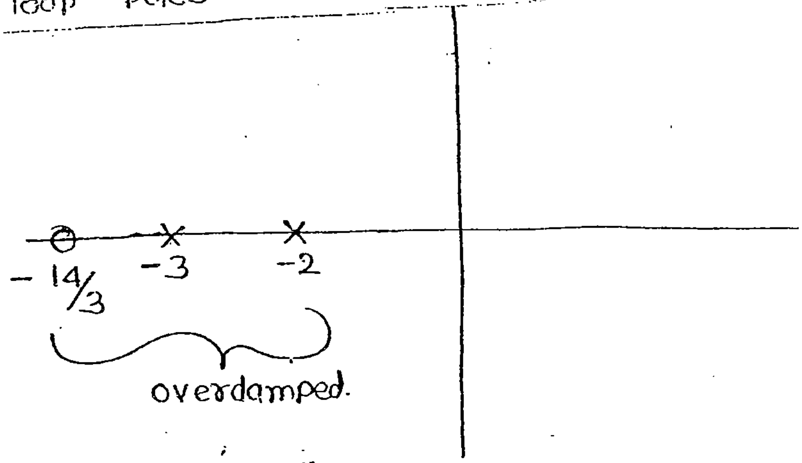
$s=-2,-3$



S (overdamp)

* R.L.D. of above model is

*** Whenever the T.F. not consist the any system gain parameter then RLD is nothing but closed loop poles and closed loop zero.



* Solution to the state equation

$\dot{x} = AX + BU$ (Non- Homogeneous state eqn).

method-1 L.T. method →

$sX(s) - X(0) = AX(s) + BU(s)$.

$sX(s) - AX(s) = X(0) + BU(s)$.

$X(s)[sI - A] = X(0) + BU(s)$

$$X(s) = (sI-A)^{-1} x(0) + (sI-A)^{-1} B U(s)$$

$$\longrightarrow X(t) = \underbrace{L^{-1}\{(sI-A)^{-1} x(0)\}}_{ZIR} + \underbrace{L^{-1}\{(sI-A)^{-1} BU(s)\}}_{ZSR}$$

ZIR → due to initial cond.

ZSR → always due to i/p

② classical method

$$x(t) = \underbrace{e^{AT} x(0)}_{ZIR} + \int_0^t e^{A(t-\tau)} B\, u(\tau)\, d\tau \quad \text{—— ②}$$

$\phi(t-\tau)$

Compare

⟹ ZIR

$$\boxed{\phi(t) = e^{At} = L^{-1}[(sI-A)^{-1}]}$$

(STATE TRANSITION-MATRIX)

⟹ Compare ZSR

$$\int_0^t \phi(t-\tau) B u(\tau) d\tau = L^{-1}[\phi(s)\, B\, U(s)]$$

$$\boxed{x(t) = e^{At} x(0) + L^{-1}[\phi(s) B U(s)]} \quad \text{—— ③}$$

Homogeneous state equation (u=0)

$$\dot{X} = AX$$

$$X(t) = ZIR = e^{At} x(0)$$

* Properties of (STM)

① $\phi(0) = I$ (Identity matrix)

② $\phi^K(t) = (e^{At})^K = e^{A(Kt)} = \phi(Kt)$ $\quad \boxed{\phi^{-1}(t) = \phi(-t}$

③ $\phi(t_1+t_2) = \phi(t_1)\cdot\phi(t_2)$.

④ $\phi(t_2-t_1)\cdot\phi(t_1-t_0) = \phi(t_2-t_0)$

* PROBLEMS

obtain complete time Response to the given system.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x \qquad X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad Y = [1 \ \ -1] X$$

The given State model is Homogeneous. ∵ I/P = 0.

Then the Solution is

$$x(t) = e^{At} x(0) = \phi(t)\, x(0)$$

STM

$$\phi(t) = e^{At} = L^{-1}\left[(SI-A)^{-1}\right] = L^{-1}\begin{bmatrix} \frac{S}{S^2+2} & \frac{1}{S^2+2} \\ \frac{-2}{S^2+2} & \frac{S}{S^2+2} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}}_{\phi(t)} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x(0)}$$

$$ZIR = \phi(t)\, X(0) = \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

the Complete time Response means $y(t)$

$$Y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$\boxed{Y = \frac{3}{\sqrt{2}}\sin\sqrt{2}t}$$

* obtain the Complete time Response for unit Step I/P of the System given by

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u \qquad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & 1 \end{bmatrix} X.$$

given State model – Non Homogeneous.

solution = ZIR + ZSR.

ZIR $= e^{At} x(0)$

$$(SI-A) = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S^2+3S+2}$$

$$L^{-1}\left\{\begin{matrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{matrix}\right\}$$

$$\Rightarrow L^{-1}\begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\phi(t) \Rightarrow \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$ZIR = \phi(t)\,X(0) = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$ZSR = L^{-1}\{\phi(s)\,B\,U(s)\}$$

$$= L^{-1}\begin{bmatrix} \frac{(s+3)}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}\underbrace{\begin{bmatrix} 0 \\ 5 \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix}}_{\begin{bmatrix} 0 \\ 5/s \end{bmatrix}}$$

$$ZSR = L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{5}{(s+1)(s+2)} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} - 5e^{-t} + \frac{5}{2}e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

$$X(t) = ZIR + ZSR = \begin{bmatrix} 5/2 - 3e^{-t} + 3/2\, e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}$$

Put in

$$Y = [0 \quad 1]\, X(t)$$

$$\boxed{Y = 3\left[e^{-t} - e^{-2t}\right]}$$

* Controllability and ...

* A system is said to be controllable if it is possible to transfer the initial states to the desired state in a finite time interval, by the controlled i/p.

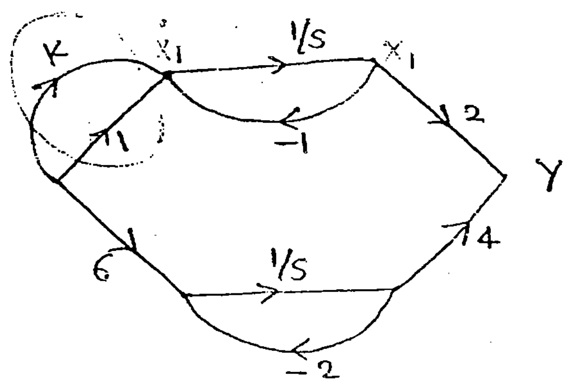
* If SFG is given to check the controllability then observe the path from i/p to each and every state variable. If the path is exist then the system is controllable. [If no] path is exist then the system uncontrollable.

* Find the K value to [become] the system uncontrollable [for] the following system.

$K+1=0$

$[K=-1]$

to become uncontrollable



(No path exist from i/p

Kalaman's test Controllability

$$Q_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{(n-1)}B \end{bmatrix}$$

$$\left\{ \begin{array}{l} \text{Rank of } Q_c = \text{Rank of } A. \\ |Q_c| \neq 0 \end{array} \right\}$$

Check the controllability

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 4}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$n = 3.$

$$Q_c = \begin{array}{ccc} B & AB & A^2B \\ \end{array}$$
$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|Q_c| = -1$$
$$\neq 0 \quad \text{controllable}$$

Observability –

A system is said to be observable, if it is possible to determin the initial states of the system to the desired state in a finite time interval.

** KALAMAN'S TEST FOR OBSERVILITY :-

$$Q_o = [C^T, \; A^T C^T, \; (A^T)^2 C^T \; \ldots \; (A^T)^{n-1} C^T] = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^n \end{bmatrix}$$

observable

Rank of Q_o = Rank of A

$$|Q_o| \neq 0$$

(1) $\dot{X} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u. \qquad Y = [1 \quad 1] X.$

(2)
$$\dot{x}_1 = -2x_1 + x_2 + u$$
$$\dot{x}_2 = -x_2 + u$$
$$Y = x_1 + x_2$$

Ans (1)

$$Q_c = \begin{array}{cc} B & AB \end{array}$$
$$Q_c = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \qquad |Q_c| = 0 \quad (\text{Not Controllable})$$

$$Q_o = \begin{array}{c} C \\ CA \end{array} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \qquad |Q_o| = 0 \quad (\text{Not observable}).$$

Ans (2)

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = [1 \quad 1]$$

$$Q_o = \begin{array}{c} C \\ CA \end{array} \begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix} \neq 0 \quad (\text{observable})$$

$$Q_c = \begin{bmatrix} B & AB \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \qquad |Q_c| \neq 0 \quad \text{(Not Controllable)}$$

Compensator →

A compensator is a electrical n/w which adds one finite pole, one finite zero to the system.

① Lead

② Lag

③ Lag – lead.

① Lead compensator –

① when sinusoidal i/p is applied to the n/w it produce the sinusoidal steady state o/p having phase lead with input. then n/w is called Lead compensator.

* A lead compensator speed up the Transient Response and increase the margin / for system stability and also it increases the error constant.

② Lag compensator –

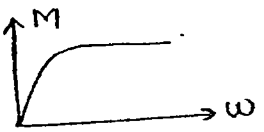
If the steady state o/p has phase lag then the n/w is called lag compensator. Improves the steady state behaviour the lag compensator without affecting its transient response

③ Lag - lead Compensator –

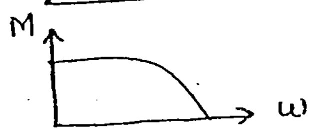
In lag - lead compensator Both phase lag and lead occurs. But in different frequency region. the phase lag at low freq. and phase lead at High freq. Improves the Both transient and the lag-lead compensator steady state performance

Filter

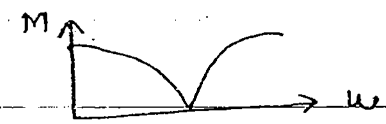
LEAD ——— HIGH



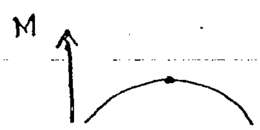
LAG ——— LOW



LAG - LEAD ——— BAND STOP



LEAD - LAG ——— BANDPASS.

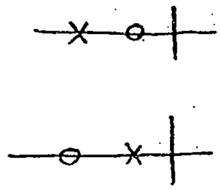


pole zero order

(1)

+ve angle given by zeros

-ve angle given by poles



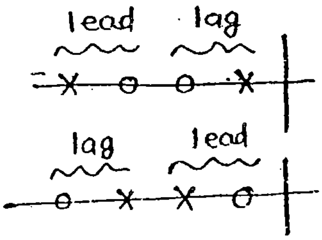
$TF = \dfrac{1+\tau s}{1+\alpha \tau s}$

$\alpha < 1$ (Lead (0.05

$\alpha > 1$ (lag) (≥ 10)

$\tau_{lag} > \tau_{lead}$

$\tau_{lead} > \tau_{lag}$

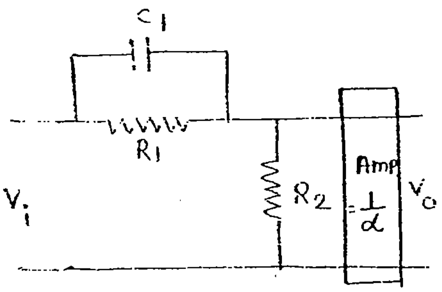


$$\ast \quad \boxed{\omega_{max\ phase\ \binom{lag}{lead}} = \frac{1}{\tau\sqrt{\alpha}}}$$

$$\ast \quad \phi_{max\,(lead)}\Big|_{\omega=\omega_{max}} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\phi_{max\,(lag)}\ \omega=\omega_{max} = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right)$$

Lead



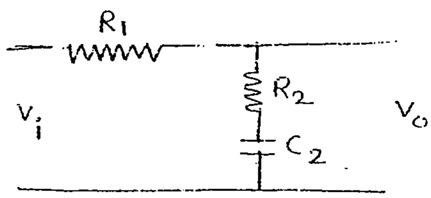
$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{\left(R_1 \| \frac{1}{sC_1}\right) + R_2}$$

$$= \frac{R_2(1+sC_1R_1)}{R_1 + R_2(1+sC_1R_1)}$$

$$= \frac{R_2(1+sC_1R_1)}{R_1+R_2+sC_1R_1R_2}$$

$$= \frac{R_2(1+s(C_1R_1))}{(R_1+R_2)\left(1+\frac{R_2}{R_1+R_2}sC_1R_1\right)}$$

Lag.



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC_2}{R_1+R_2+1/sC_2}$$

$$= \frac{sC_2R_2+1}{1+sC_2R_2\cdot\frac{(R_1+R_2)}{R_2}}$$

let $\alpha =$ lag const.

$$\frac{R_1+R_2}{R_2} > 1$$

$\tau =$ lag time const. $\simeq R_2C_2$

$$\frac{V_o(s)}{V_i(s)} = \frac{1+\tau s}{1+\alpha \tau s}$$

* let α = lead constant

$$= \frac{R_2}{R_1+R_2} < 1$$

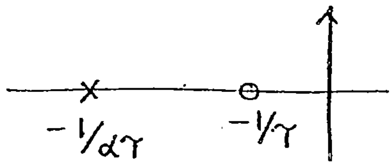
$\tau \rightarrow$ lead time Const. $= R_1C_1$

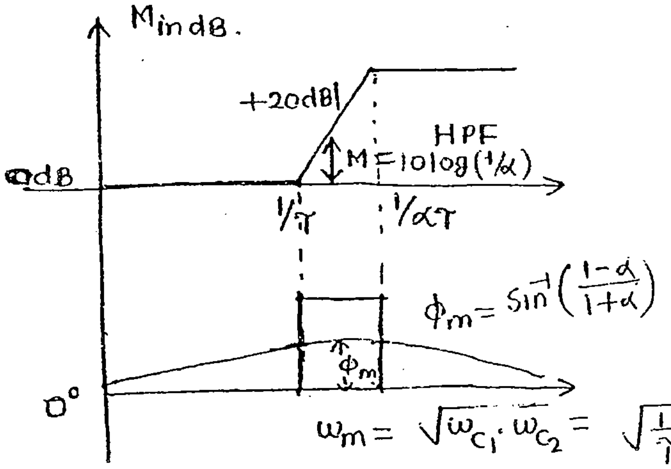
$$\frac{V_o(S)}{V_i(S)} = \frac{\alpha(1+\tau s)}{(1+\alpha\tau_s)}$$

* main Disadvantage of lead Compensator is, It creates the attenuation in the system, to overcome the attenuation we require to connect a Amplifier with a gain of $1/\alpha$ in series to the system.

By adding the amplifier, increases the cost and space (Area) of the System.

$$S_p = -\frac{1}{\alpha\tau} \text{ (Pole)}$$

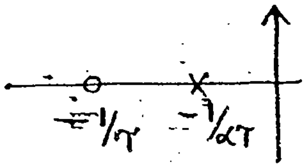


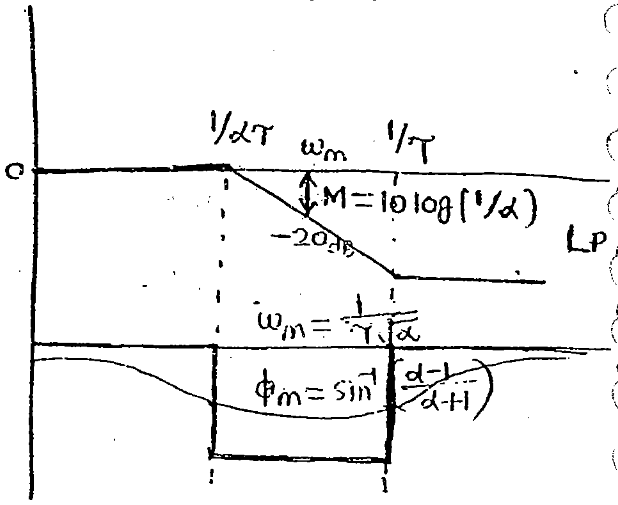


$$\omega_m = \sqrt{\omega_{c_1}\cdot\omega_{c_2}} = \sqrt{\frac{1}{\tau}\cdot\frac{1}{\alpha\tau}}$$

$$= \frac{1}{\tau\sqrt{\alpha}}$$

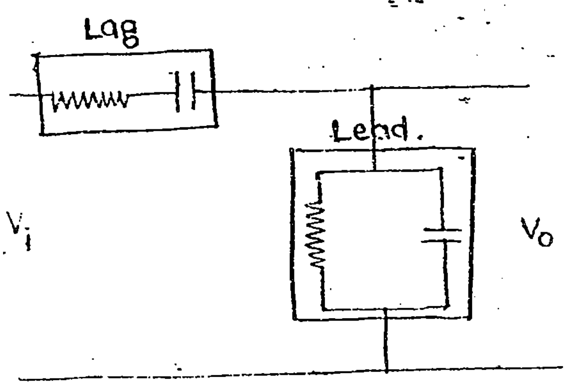
LAG





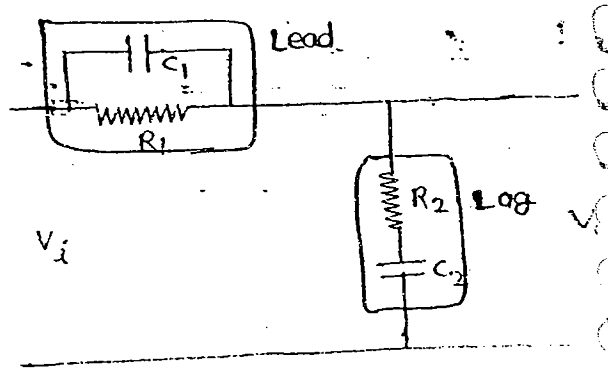
The lead or lag compensator give the max. phase lead or lag angle by $+60°$ or $-60°$.

Lag – lead



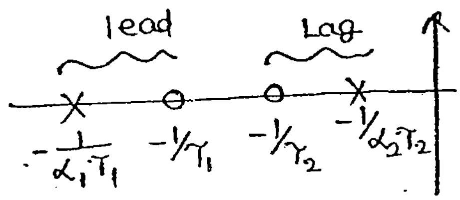
$(\tau_{Lead} \not< \tau_{Lag})$

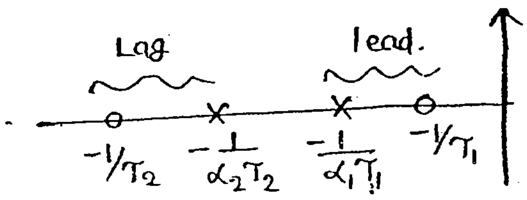
Lead – Lag.

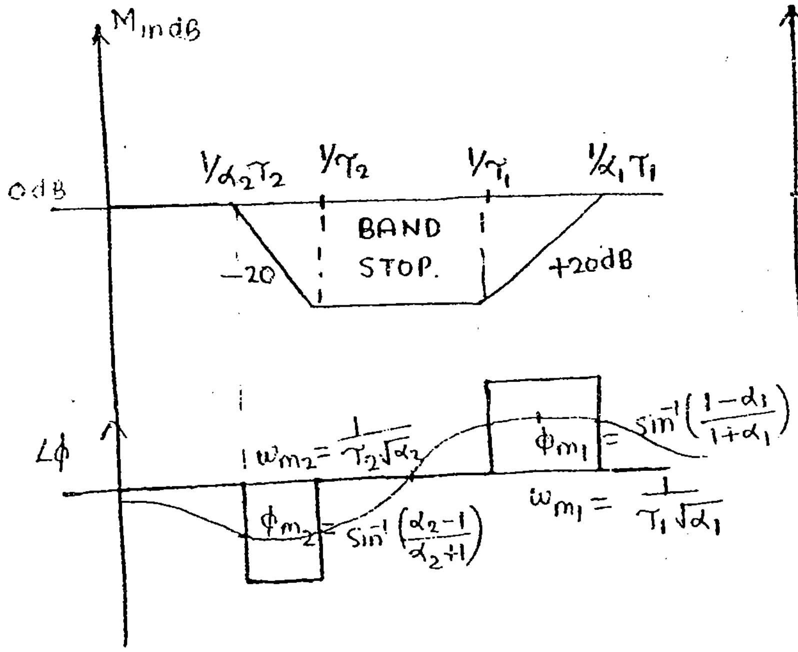


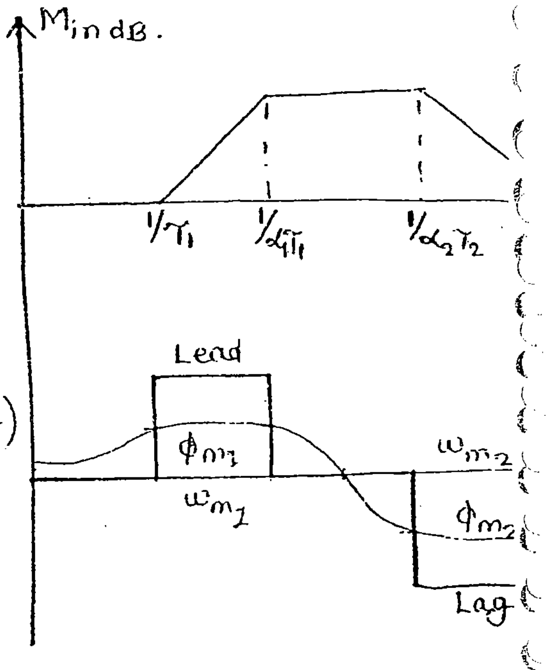
$(\tau_{Lead} > \tau_{Lag})$

$$\frac{V_o(s)}{V_i(s)} = \underbrace{\left(\frac{1+\tau_1 s}{1+\alpha_1\tau_1 s}\right)}_{\text{Lead}} \underbrace{\left(\frac{1+\tau_2 s}{1+\alpha_2\tau_2 s}\right)}_{\text{Lag}}$$



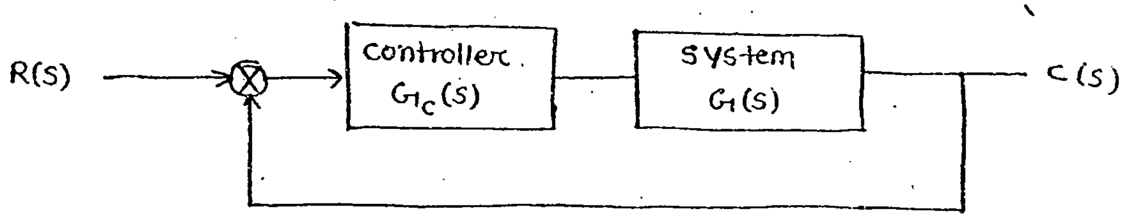






PID Controller

A Controller is a device which is used to control the transients and steady state as per the requirement. The Best System demand less settling time, less t_r, less e_{ss}, less m_p to get the above requirements, we require to use the Controllers. the Block diagram with controller is shown in the fig



Proportional Controller

Purpose

* to Change the Transient response as for the requirement.
* The Proportional Controller can not eliminate the complete error. (Error x Controller)
* The T.F. of Proportional Controller $= K_p$

Let

$$G(S)\Big|_{\text{without Controller}} = \frac{1}{S(S+10)}$$

$$CLTF = \frac{1}{S^2+10S+1}$$

$\omega_n = 1$,
$2\zeta\omega_n = 10$
$\boxed{\zeta = 5}$ (overdamped)

$$G(S)\Big|_{\text{With Controller}} = \frac{K_p}{S(S+10)}$$

$$CLTF = \frac{K_p}{S^2+10S+K_p}$$

$K_p = 100$ $\quad 2\zeta\omega_n = 10$
$\omega_n = 10$ $\quad (\zeta = 0.5)$
underdamp.

by Selecting proper value of K_p we can get Required Transient performance

as K_p value ↑ the ζ ↓ Hence % m_p ↑ So System less stab

* PID Controller →

Purpose

* To improve system stability and decrease e_{ss}.
* T.F. of PID controller = $\left(K_P + \frac{K_I}{S} + K_D S\right)$

$$= \frac{K_D S^2 + K_P S + K_I}{S}$$

* The PID controller added one pole at origin which increase the type Hence $e_{ss} \downarrow$
* The PID controller added two finite zeros in Left Hand Side, one finite zero, avoid the affect on stability other zero improves the stability.

$\boxed{\text{For example}}$ →

$$G(S)\Big|_{w/c} = \frac{1}{S^2(S+10)} \qquad \text{Type-2}$$

$$S^3 + 10S^2 + 1 = 0 \quad \text{US}$$

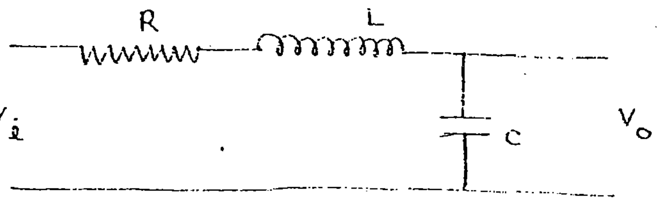
Stability improved.

$$G(S)\Big|_{wc} = \frac{K_D S^2 + K_P S + K_I}{S^3(S+10)} \qquad \text{Type-3.} \qquad e_{ss} \downarrow\downarrow$$

$$S^4 + 10S^3 + K_D S^2 + K_P S + K_I = 0 \quad \text{S}$$

Frequency domain Specification

general Frequency Response of any R-L-C ckt. as shown in figure.



$$\frac{V_o(S)}{V_i(S)} = \frac{1}{S^2 LC + SCR + 1}$$

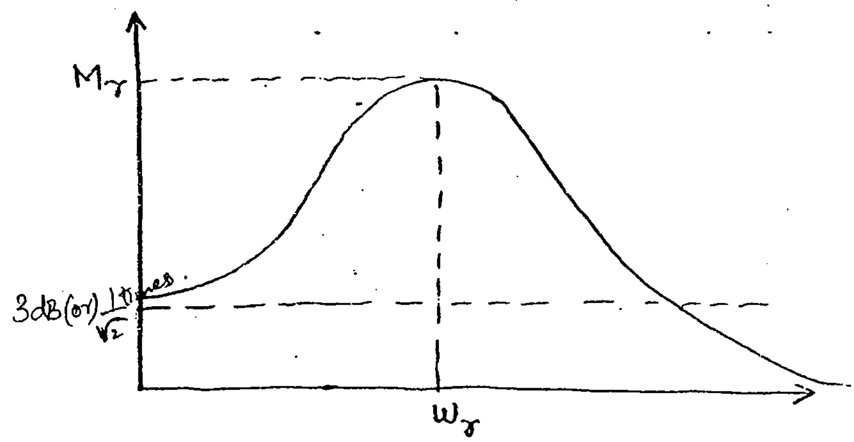
$$= \frac{1/LC}{S^2 + S\,\frac{R}{L} + \frac{1}{LC}} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$

$$2\zeta\omega_n = \frac{R}{L}$$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

Imp. $\left\{\begin{array}{l} R=0 \\ \zeta=0 \end{array}\right. \rightarrow$ undamped nature

** $\boxed{Q = \frac{1}{2\zeta}} = \frac{1}{R}\sqrt{\frac{L}{C}}$
*



Resonant frequency
The freq. at which the Resonant peak or max. magnitude occur.

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} \text{ rad/sec.} \quad (\zeta < \tfrac{1}{2})$$

Resonant peak –
It is a max. magnitude occurs at Resonant frequency

$$M_r = \frac{1}{2\zeta\sqrt{1-2\zeta^2}}$$

Bandwidth

$$BW = \omega_n\sqrt{1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}}$$

It is the Range of frequencies at which the magnitude dropped by 3 dB. or 0.707 from M_r is called. BW.

* Sensitivity
→ It is used to describe the Relative variation in parameter like G(S), H(S)
→ sensitivity of T.F. w.r.t. variations in G(S) is denoted as

$$S_G^T = \frac{\text{\% of change in T.F.}}{\text{\% of change in } G(S)}$$

$$= \frac{\partial T/T}{\partial G/G} = \left(\frac{G}{T}\right)\left(\frac{\partial T}{\partial G}\right)$$

Similarly,

$$S_H^T = \left(\frac{H}{T}\right)\left(\frac{\partial T}{\partial H}\right)$$

Integral Controller (Reset controller)

Purpose - * to decrease the e_{ss}.

* the integral controller can eliminate the complete error which is left by the proportional controller

T.F. of Integral controller = K_i/s

the integral controller added one pole at origin which increases the type, as type ↑, e_{ss} ↓ But System Stability is affected.

For example

without controller - $G(s) = \frac{1}{s(s+10)}$ type - 1

CE $s^2 + 10s + 1 = 0$ (S)

with controller -

Stability affected.

$$G(s) = \frac{K_I}{s^2(s+10)}$$ type - (2)

CE ⇒ $s^3 + 10s^2 + K_I = 0$ (U.S)

(missing)

Derivative controller (Rate controller)

Purpose -

* to Improve the Stability
* The T.F. of derivative controller is = $K_D s$

The Best example for Derivative controller is Tachometer. $\boxed{K_t s}$

The Derivative controller added 1 zero at origin Hence type is decreases, as type decrease Stability Improve but e_{ss} ↑.

* without controller

$$G(s) = \frac{1}{s^2(s+10)}$$ Type = 2

$$CE. = s^3 + 10s^2 + 1$$ (US)

* with controller

(S) Improved

$$G(s) = \frac{K_D \cancel{s}}{s^{\cancel{2}}(s+10)}$$

$$= s^2 + 10s + K_D = 0$$ (Stable)

B

** PI controller

Purpose-

* to decrease e_{ss} without affecting stability.
* T.F. of PI controller =

$$\left(K_p + \frac{K_I}{S}\right) = \frac{SK_p + K_I}{S}$$

The P.I. controller added one pole at origin. Hence type is increases, so e_{ss} is decreases.
The P.I. controller added one finite zero at the left Hand side which avoid the effect on system stability.

For example

without controller $\quad G(S) = \dfrac{1}{S(S+10)} \quad$ Type = 1

CE → $\quad S^2 + 10S + 1 = 0 \quad$ (S)

* with controller $\quad G(S) = \dfrac{(SK_p + K_I)}{S^2(S+10)} \quad$ Type = 2 $\quad e_{ss}\downarrow\downarrow$

not affected

*

* CE ⇒ $\quad S^3 + 10S^2 + SK_p + K_I = 0 \quad$ (S)

P-D controller

Purpose

* to improve stability without affecting e_{ss}.
* The T.F. of PD controller = $K_p + K_D S$.
* PD controller added one finite zero in Left Hand Side which improves the System Stability.

The PD controller not changes the type, Hence no change in e_{ss}. for example.

without controller $\quad G(S) = \dfrac{1}{S^2(S+10)} \quad$ type-2 $\quad$ (U.S)

$$S^3 + 10S^2 + 1 = 0$$

with controller. $\quad G(S) = \dfrac{(SK_D + K_p)}{S^2(S+10)} \quad$ Type-2 (No affect on e_{ss})

CE - $\quad S^3 + 10S^2 + SK_D + K_p = 0 \quad$ (S)

Sensitivity of system w.r.t.

(1) G(s)

(2) H(s).

$$T = \frac{G}{1+GH}$$

$$S_G^T = \frac{\cancel{G}}{\left(\frac{\cancel{G}}{\cancel{1+GH}}\right)} \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{1}{1+GH}$$

$$S_H^T = \frac{H}{\left(\frac{G}{1+GH}\right)} \left(\frac{0 - G \times G}{(1+GH)^2}\right) = \frac{-GH}{1+GH}$$

*** $\boxed{S_H^T > S_G^T}$

Feedback n/w is more sensitive than Forward path gain.

<u>OL system</u> $(T = G)$

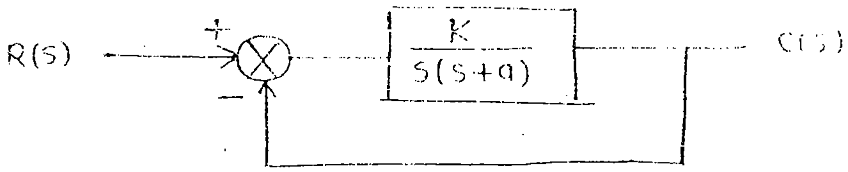
$$S_G^T = \left(\frac{G}{G}\right) = 1$$

OL system is more sensitive than CL system

(*) Find the sensitivity of given system w.r.t. variation in

→ a

→ K



<u>CLTF</u>

$$T = \frac{K}{s^2+as+K}$$

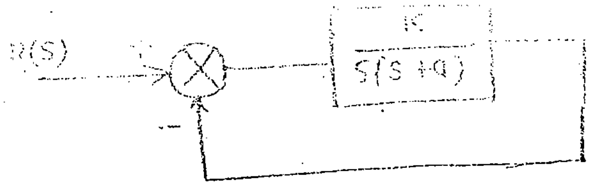
$$S_a^T = \frac{\partial T/T}{\partial a/a} = \left(\frac{a}{T}\right)\left(\frac{\partial T}{\partial a}\right) \Rightarrow \frac{a}{\left(\frac{K}{s^2+as+K}\right)} \left\{\frac{0 - Ks}{(s^2+as+K)^2}\right.$$

$$= \frac{-as}{s^2+as+K}$$

$$S_K^T = \frac{\partial T/T}{\partial K/K} = \frac{K}{\left(\frac{K}{s^2+as+K}\right)} \left\{\frac{s^2+as+K-K}{(s^2+as+K)^2}\right\}$$

$$= \frac{s(s+a)}{s^2+as+K}$$

Find the sensitivity of e_{ss} [illegible]



$$G(s) = \frac{K}{s(s+a)}$$

$$e_{ss} = \frac{A}{K} = \frac{1}{K/a} = a/K$$

$$S_K^{e_{ss}} = \frac{\partial e_{ss}/e_{ss}}{\partial K/K} = \left(\frac{K}{e_{ss}}\right)\left(\frac{\partial e_{ss}}{\partial K}\right)$$

$$= \frac{K^2}{a}\left(\frac{-a}{K^2}\right) = -1$$

$$S_a^{e_{ss}} = \frac{\partial e_{ss}/e_{ss}}{\partial a/a} = \left(\frac{a}{e_{ss}}\right)\left(\frac{\partial e_{ss}}{\partial a}\right) = 1$$

98430427[illegible]

om. [illegible] @ gmail.com